

# Introduction to Planetary Sciences

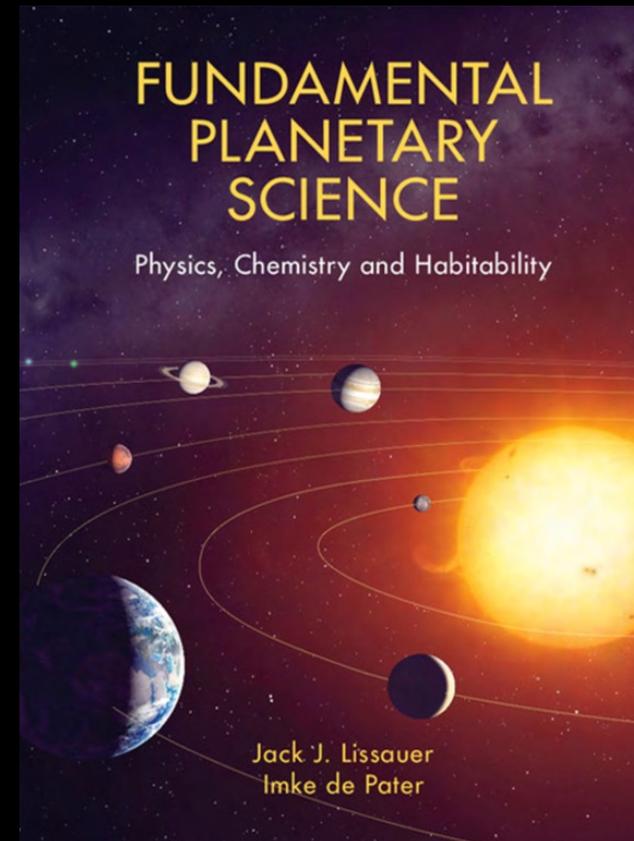
## Textbook

*Fundamental Planetary Science: Physics, Chemistry and Habitability*

By Lissauer J.J., Pater I.d, Cambridge University Press (2019)

## Instructor

- Ruobing Dong / rbdong@pku.edu.cn
- Office: KIAA 313
- Office hours: by appointment



# Introduction to Planetary Sciences

## Course Website

<https://www.ruobingdong.com/introductiontoplanetarysciences>

Ruobing Dong's Group   Home   Group   Research   Publications   Media   Teaching

### Introduction to Planetary Sciences

#### Lecture Notes

[Textbook errata](#)

#### Assignments

[Assignment 1 \(due 23:59, March 20, 2026\)](#)

[Assignment 2](#)

[Assignment 3](#)

[Assignment 4](#)

#### Chapters not covered in this course:

- All the chapters marked by "\*" in the book
- 2.5 / 2.6 / 2.7 / 3.1.2 / 3.1.3 / 3.1.4 / 4.5.3 / 4.5.4 / 4.6.1 / 4.6.2 / 5.4 / 5.5.2 / 5.6

#### Final Pre Sign Up Sheet

[\[Tencent Docs\] Intro to Planetary Sciences Final Pre Schedule](#)  
<https://docs.qq.com/sheet/DTVZwU3JfSldPem1S?tab=BB08J2>



# Tentative schedule

- Assignment
- Presentation
- Final

## March

	M	T	W	T	F	S	S
9	23	24	25	26	27	28	1
10	2	3	4	5	6	7	8
11	9	10	11	12	13	14	15
12	16	17	18	19	20	21	22
13	23	24	25	26	27	28	29
14	30	31	1	2	3	4	5

## April

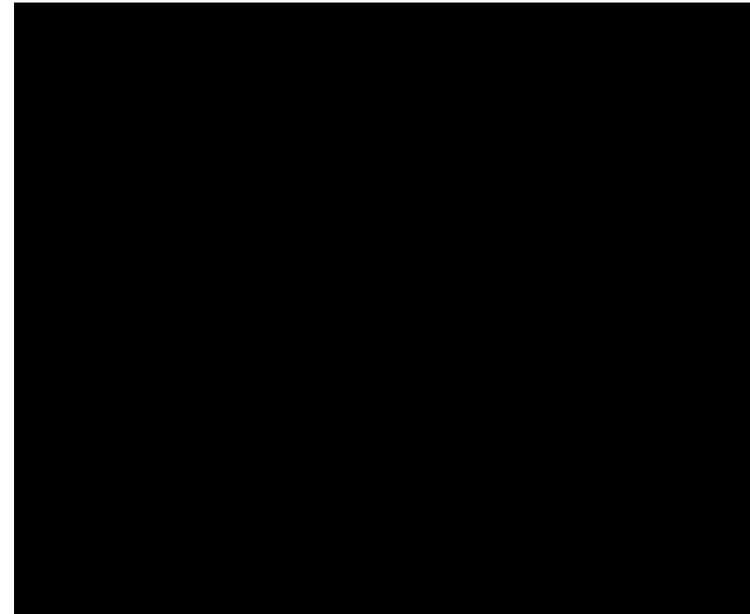
	M	T	W	T	F	S	S
14	30	31	1	2	3	4	5
15	6	7	8	9	10	11	12
16	13	14	15	16	17	18	19
17	20	21	22	23	24	25	26
18	27	28	29	30	1	2	3
19	4	5	6	7	8	9	10

## May

	M	T	W	T	F	S	S
18	27	28	29	30	1	2	3
19	4	5	6	7	8	9	10
20	11	12	13	14	15	16	17
21	18	19	20	21	22	23	24
22	25	26	27	28	29	30	31
23	1	2	3	4	5	6	7

## June

	M	T	W	T	F	S	S
23	1	2	3	4	5	6	7
24	8	9	10	11	12	13	14
25	15	16	17	18	19	20	21
26	22	23	24	25	26	27	28
27	29	30	1	2	3	4	5
28	6	7	8	9	10	11	12



# Evaluations

- Assignments – 10%
  - 4 assignments in total
  - Full credit as long as the assignment is submitted on time
- Final Presentation – 30%
  - Duration: 20 minutes
  - Topic selection deadline: end of April
  - Presentation dates: May 25 and June 1
- Final Exam – 60%
  - Problems will be drawn from the assignments

## Assignment 1

### Problem 1

Scale the entire solar system such that the diameter of the Earth is 1 cm.

1. Calculate the size of the Sun and the other planets.
2. Calculate the distances from the planets to the Sun.
3. How far is the nearest star to the Sun in this system?

### Problem 2

Lagrangian points

- a. L1, L2, and L3 are along the line joining two masses  $m_1$  and  $m_2$  in the circular restricted three body problem. Assuming  $R$  is the distance between the two main objects, and  $m_2 \ll m_1$ , find out the separation between L1 and  $m_2$ .  
*Hint 1: start from the definition of L points.*  
*Hint 2: when  $m_2 \ll m_1$ , L1 and L2 are very close to  $m_2$ .*
- b. Find out the separation between L2 and  $m_2$ .
- c. Evaluate L1 and L2 locations in the case of the Sun-Earth system. Express the results in units of both km and in Earth-Moon separation.
- d. Is L3 closer or further away from the COM than  $m_2$ ? Why?
- e. L4 (and L5) forms an equilateral triangle with  $m_1$  and  $m_2$ . Prove that it is a Lagrangian Point in the special case of  $m_2 = m_1$ .

### Problem 3

What's the time interval between two consecutive tides induced by the moon?

*Hint 1: how many tides induced by the moon between two successive moonrises?*

*Hint 2: You might want to think about time in sidereal time. If you are unfamiliar with the concept, Wikipedia and Figure 2.21 in the book might be helpful.*

### Problem 4

The sun is losing  $6 \times 10^{12}$  grams of mass every second at the moment via its solar wind and by converting mass into radiation. The Earth orbits the Sun. As the mass of the Sun decreases, the Earth is held a bit less strongly, and its orbit expands.

- a. Derive Eqn. (2.65) in the textbook. You can assume circular orbits.
- b. Evaluate the expansion rate in units of cm/yr.

*Hint: the orbital angular momentum of the Earth is conserved in this process.*

# Final Presentation (30%)

- Topic: introduce a solar system probe
  - Pick one Solar System probe that was launched after 2000 and targeted a Solar System object (e.g., Curiosity on Mars).
  - The mission must be at least partially successful and have returned scientific results, i.e., please do not choose missions that failed at launch (e.g., exploded on the launch pad).
  - You may pick from this list:  
*[https://en.wikipedia.org/wiki/List\\_of\\_Solar\\_System\\_probes](https://en.wikipedia.org/wiki/List_of_Solar_System_probes)*
- Sign-up: First come, first serve.
- Treat your presentation as a mini-lecture for the rest of the class

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<https://docs.qq.com/sheet/DTVZwU3JESIdPem1S?tab=BB08J2>

	A	B	C	D	E	F
Date		Name of the probe	Presenter	wiki link to the probe		
May 25						
May 25						
May 25						
May 25						
May 25						
June 1						
June 1						
11	June 1					
12	June 1					
13	June 1					
14	June 1					

# ASTR 255: Introduction to Planetary Sciences

The scientific study of planets, moons, and planetary systems, both solar and extra-solar, and the processes that form them.

*This course is physics heavy, and involves some amount of math*

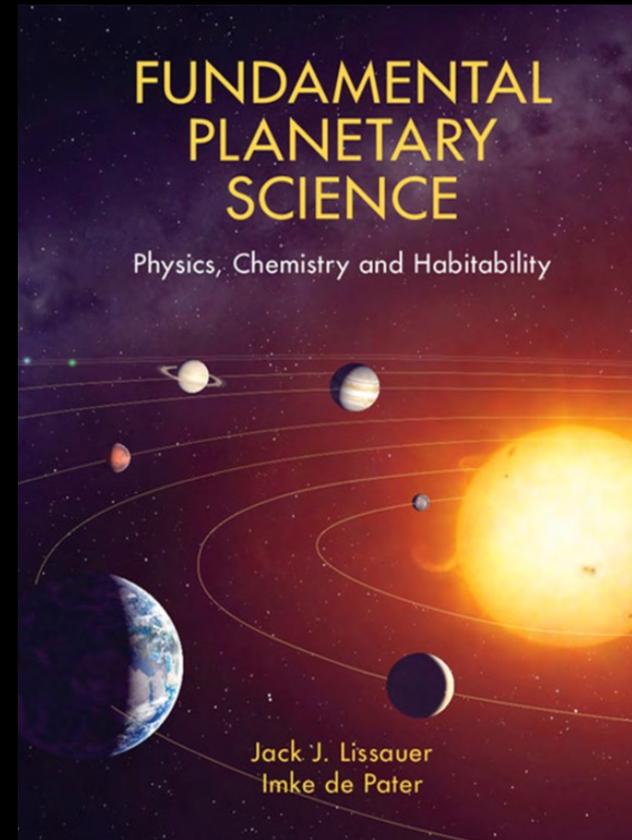
Questions?

# ASTR 255: Introduction to Planetary Sciences

*Please read the textbook  
prior to each session*

*In the classroom we will:*

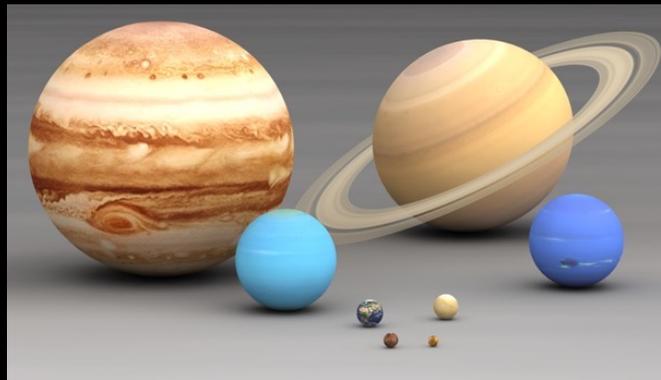
- *highlight the important points*
- *visualize things*
- *show derivations*
- *Q & A*



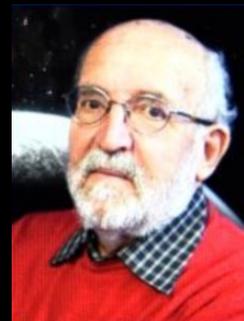
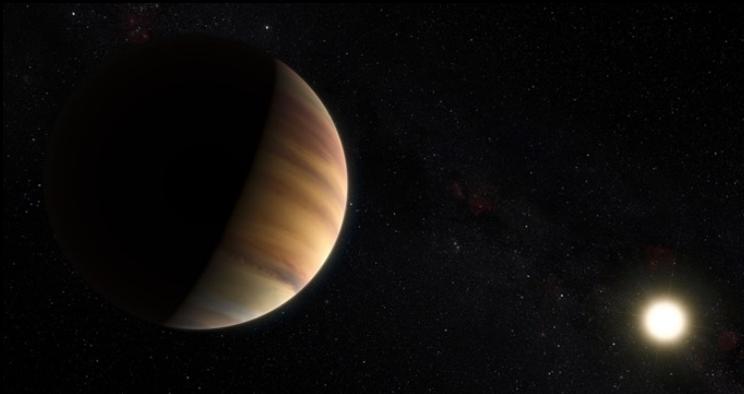
# What is a planet?

A planet is an object that

- is in orbit around the Sun
- has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (nearly round) shape,
- has cleared the neighborhood around its orbit.



- Prehistory: Mercury, Venus, Mars, Jupiter, Saturn (Copernicus Heliocentrism 16<sup>th</sup> century)
- 1781: Uranus
- 1846: Neptune
- 1995: 51 Peg b – the first exoplanet



Michel Mayor

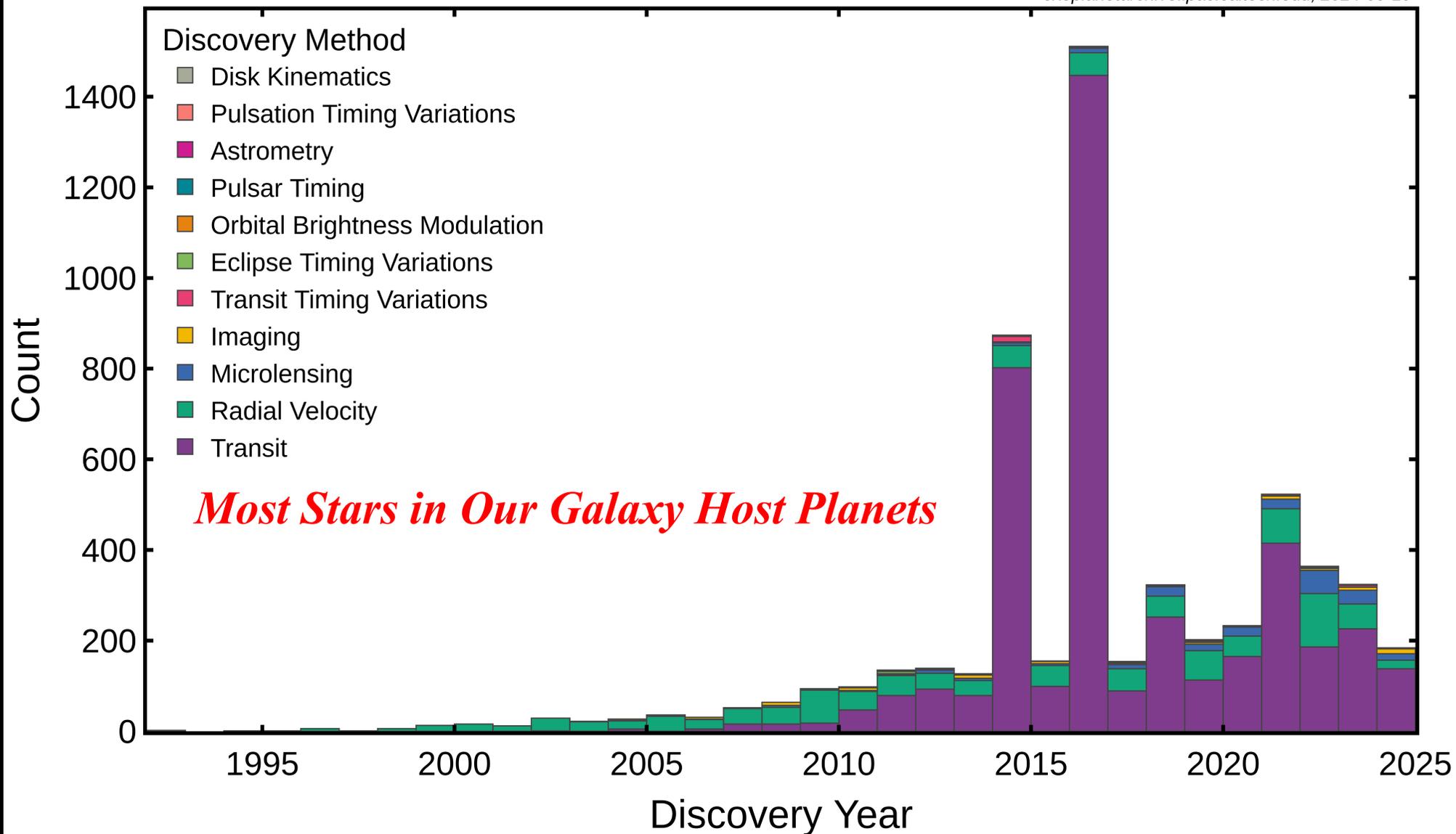


Didier Queloz

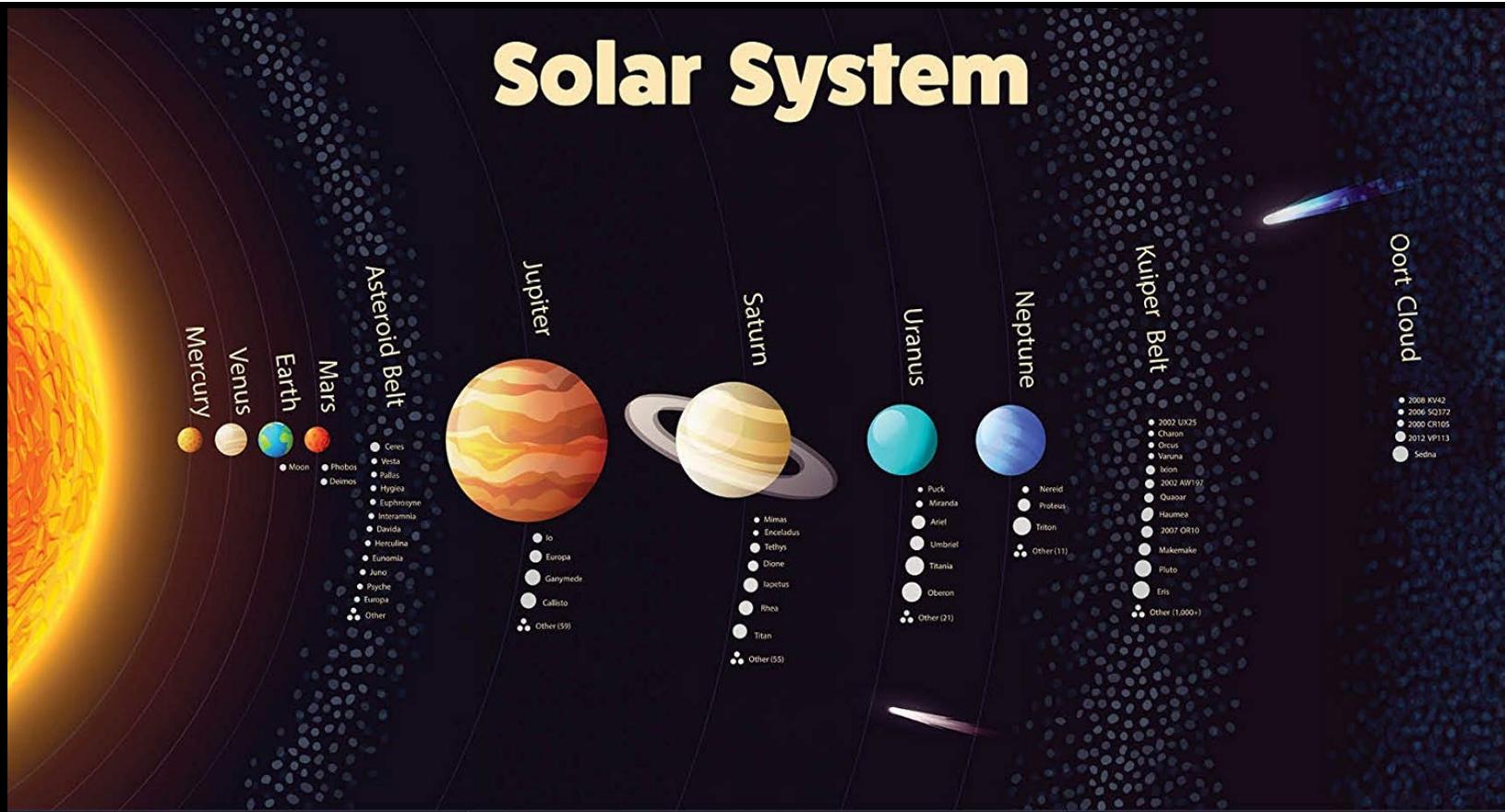
We will cover both solar system & extro-solar systems in this course

# Counts vs Discovery Year

exoplanetarchive.ipac.caltech.edu, 2024-09-19



# Solar System



- Planets
  - Terrestrial planets
  - Gas giants
  - Ice giants
- Moons and rings
- Minor objects
  - Asteroid belt
  - Kuiper belt
  - Oort cloud
  - Centaurs, comets, scattered disk objects, etc
- Dust (zodiacal light)

  
Neptune

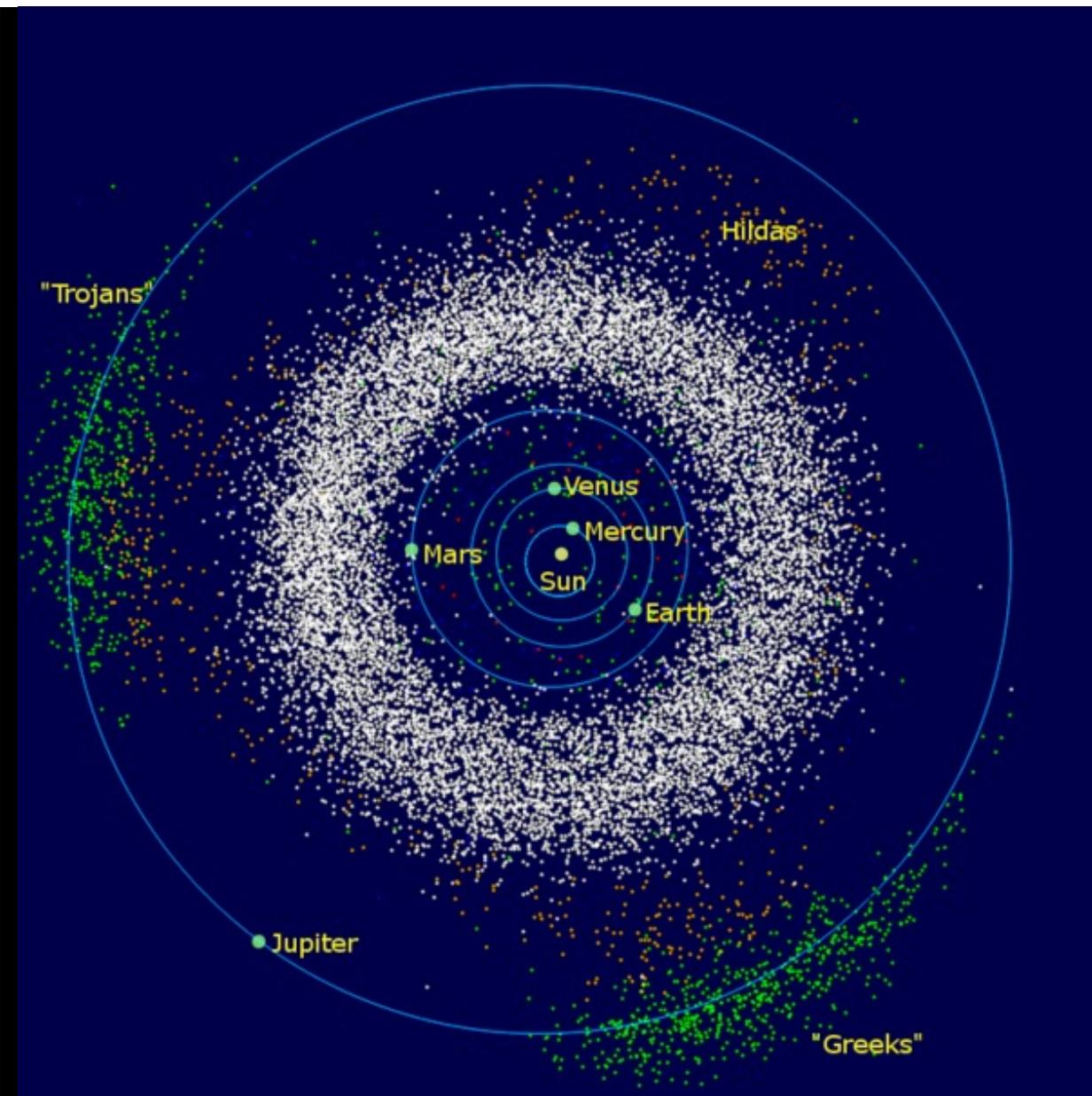
  
Uranus

  
Saturn

  
Jupiter

  
Inner planets

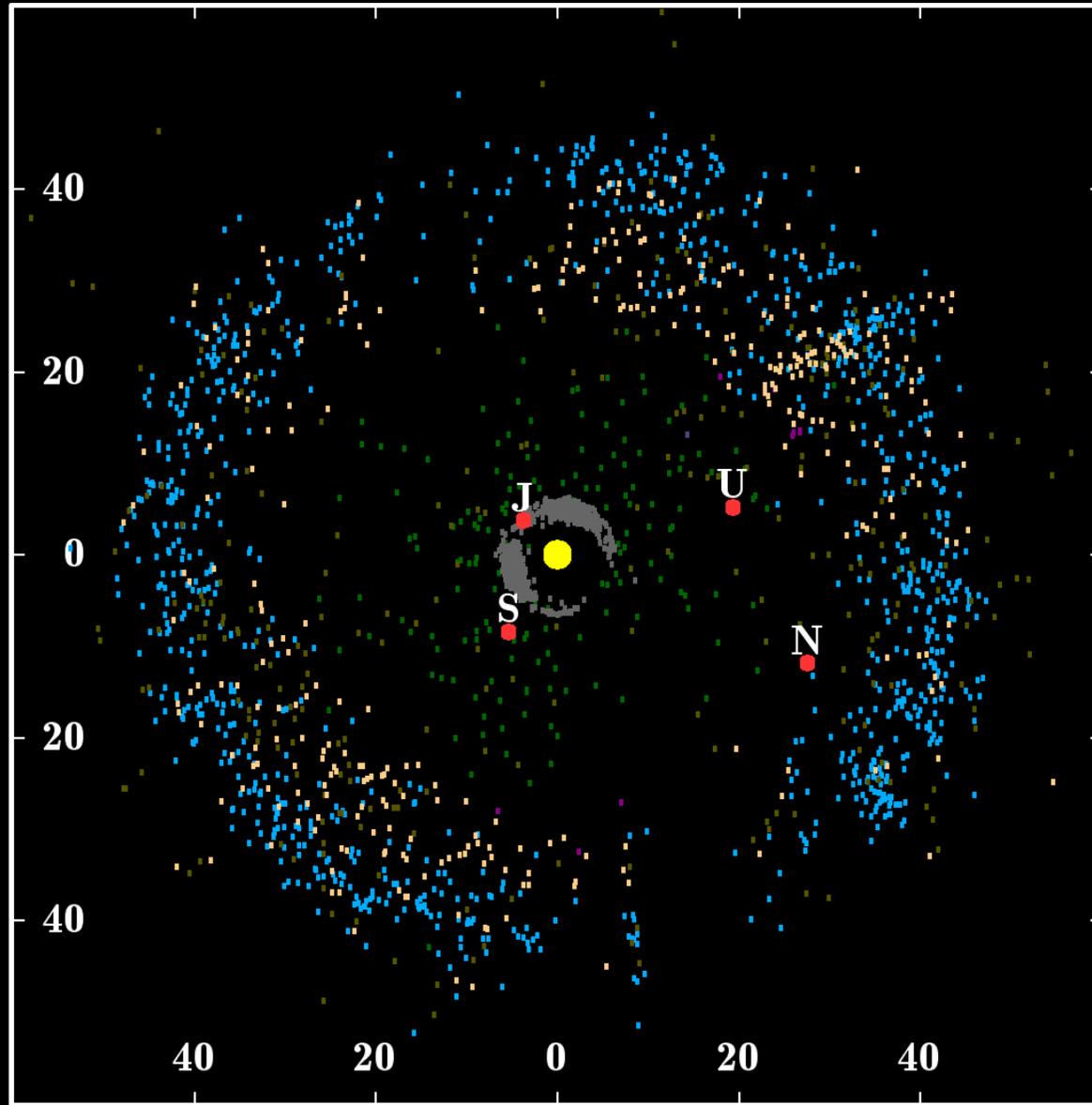
 Sun



## Inner Solar System

	Sun		<b>Asteroid belt</b>
	Jupiter trojans		Hilda asteroids (Hildas)
	Orbits of planets		Near-Earth objects (selection)

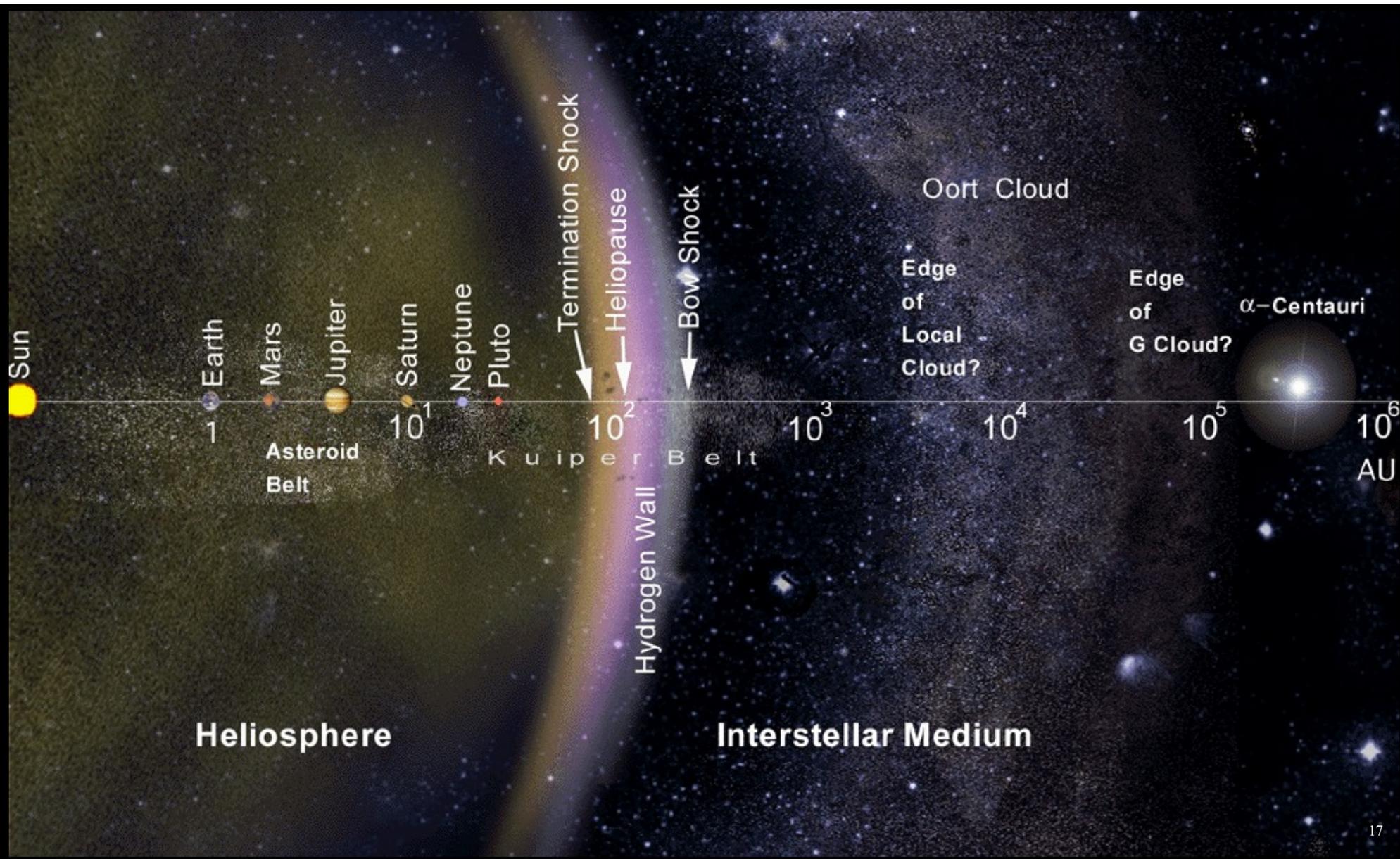
[https://en.wikipedia.org/wiki/Asteroid\\_belt](https://en.wikipedia.org/wiki/Asteroid_belt)

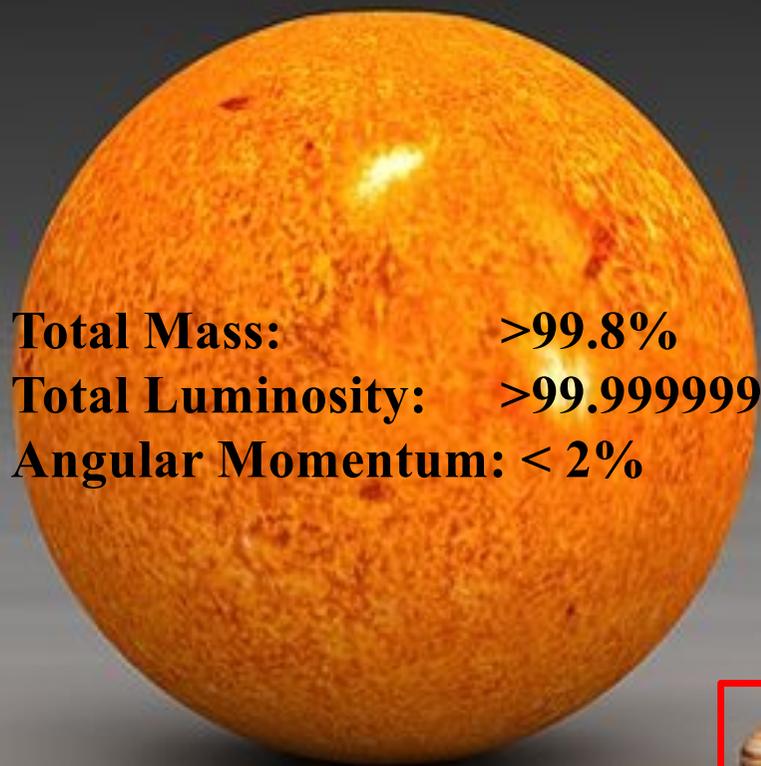


## Outer Solar System

 Sun	 Neptune trojans
 Jupiter trojans	 Resonant Kuiper belt
 Giant planets: J · S · U · N	 Classical Kuiper belt
 Centaurs	 Scattered disc

[https://en.wikipedia.org/wiki/Kuiper\\_belt](https://en.wikipedia.org/wiki/Kuiper_belt)

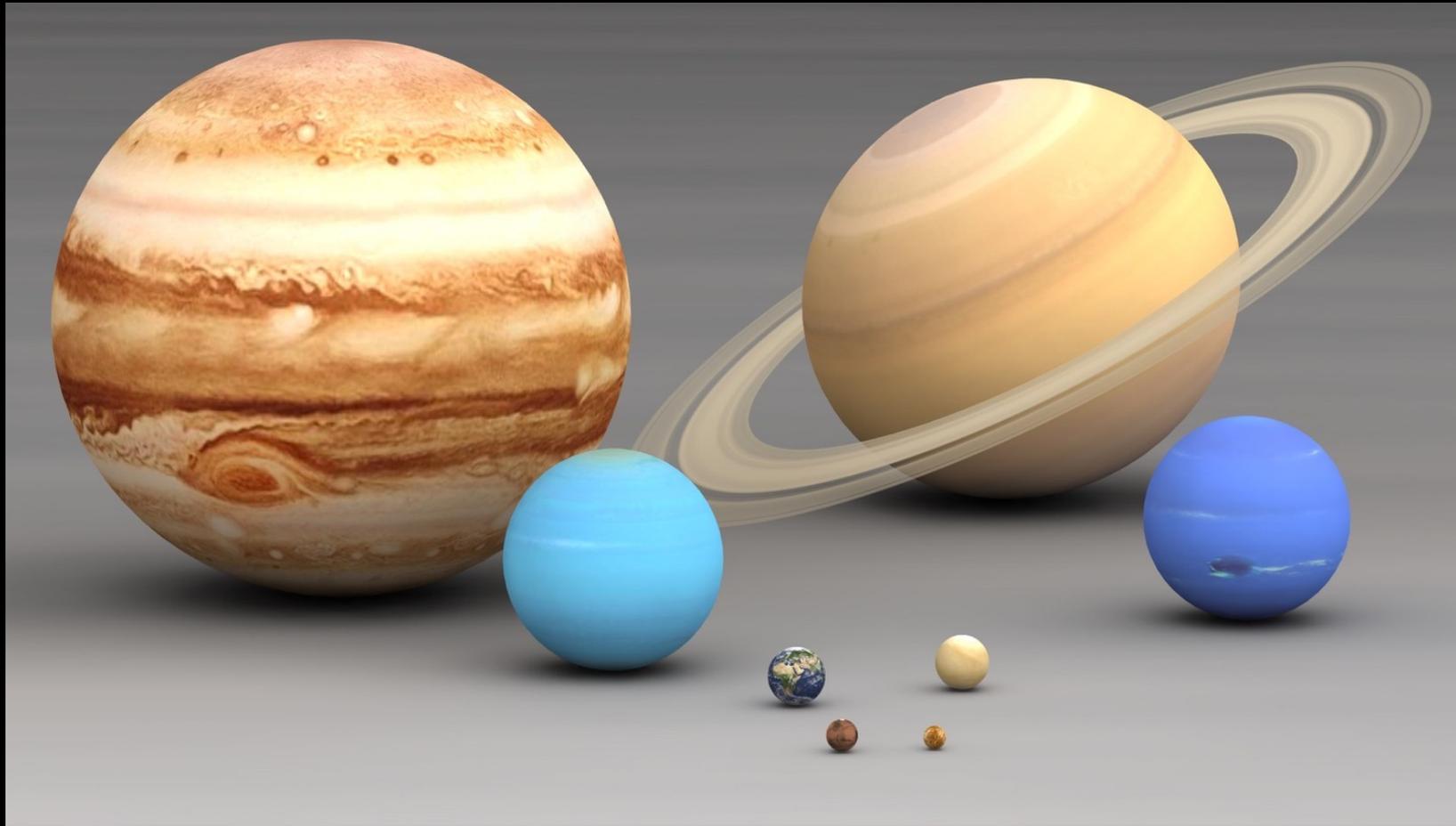




**Total Mass: >99.8%**  
**Total Luminosity: >99.999999%**  
**Angular Momentum: < 2%**

**Total Mass: < 0.2%**  
**Total Luminosity: < 0.000001%**  
**Angular Momentum: > 98%**





[https://en.wikipedia.org/wiki/Solar\\_System](https://en.wikipedia.org/wiki/Solar_System)

# Planetary Properties

C.1.4.

- (1) Orbit
- (2) Mass, distribution of mass
- (3) Size
- (4) Rotation rate and direction
- (5) Shape
- (6) Temperature
- (7) Magnetic field
- (8) Surface composition
- (9) Surface structure
- (10) Atmospheric structure and composition

## Dynamics (Chapter 2)

- Universal law of gravity

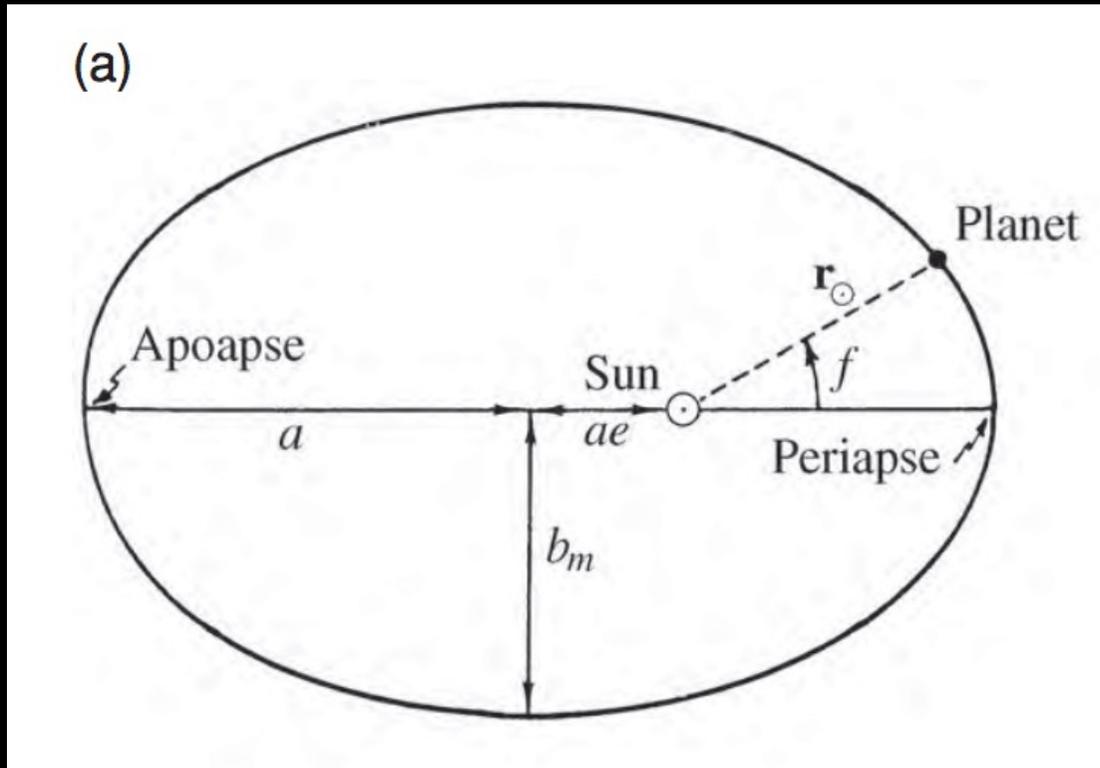
$$\mathbf{F}_{g12} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}},$$

$$\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$\hat{\mathbf{r}} \equiv \mathbf{r}/r$$

- Newton's laws
- Kepler's laws
  1. All planets move along elliptical paths with the Sun at one focus.
  2. A line connecting any given planet and the Sun sweeps out area at a constant rate.
  3. The square of a planet's orbital period about the Sun (in years) is equal to the cube of its semimajor axis (in AU).

# An Elliptical Orbit

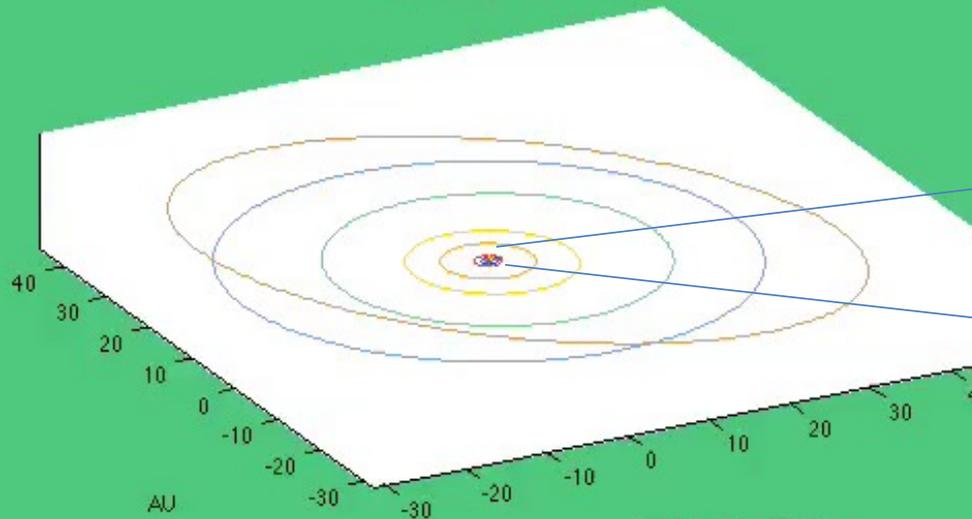


- Focus
- $a$
- $e$
- Periapse
- Apoapse

# Orbital Motions in the Solar System

## Outer Solar System

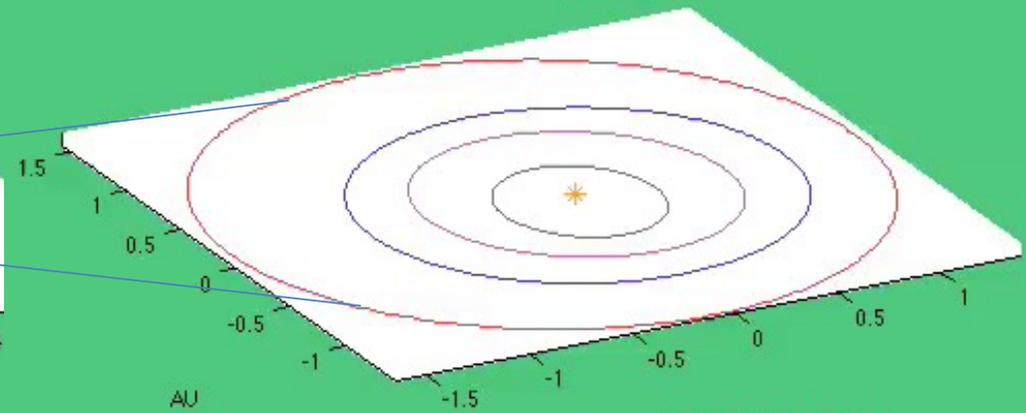
Time: 0000000 years ago



illustrated by J. Levine

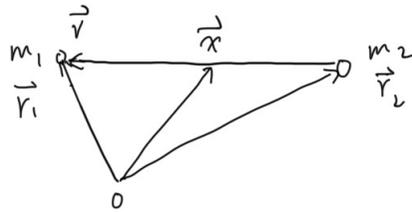
## Inner Solar System

Time: 0000000 years ago



illustrated by J. Levine

Two body problem  
 →  
 one body problem



$\vec{r} = \vec{r}_1 - \vec{r}_2$  relative position

$\vec{X} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$  COM

$m_1 \frac{d^2 \vec{r}_1}{dt^2} = - \frac{G m_1 m_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$  (2.7)

$m_2 \frac{d^2 \vec{r}_2}{dt^2} = - \frac{G m_1 m_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1)$  (2.8)

(2.7) + (2.8)  $m_1 \frac{d^2 \vec{r}_1}{dt^2} + m_2 \frac{d^2 \vec{r}_2}{dt^2} = \vec{F}_{21} + \vec{F}_{12} = 0$

$(m_1 + m_2) \frac{d}{dt^2} \left( \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} \right) = (m_1 + m_2) \frac{d^2 \vec{X}}{dt^2} = 0$

COM no acceleration

$\frac{d^2 \vec{r}}{dt^2} = \frac{d^2}{dt^2} (\vec{r}_1 - \vec{r}_2) = - \frac{G m_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2) - \left( - \frac{G m_1}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1) \right)$

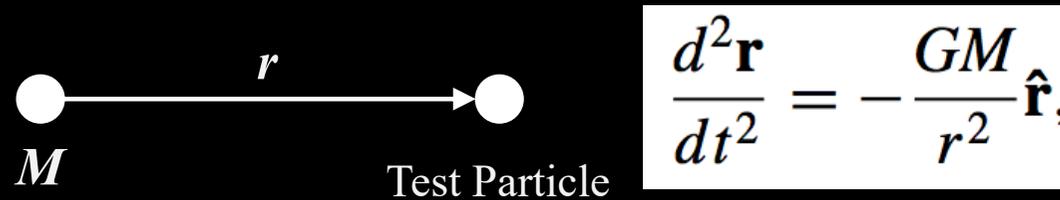
$= - \frac{G m_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2) - \frac{G m_1}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_1 - \vec{r}_2)$

$= - \frac{(m_1 + m_2)}{m} \frac{G (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3} = - \frac{G M}{r^2} \hat{r}$

# An Elliptical Orbit

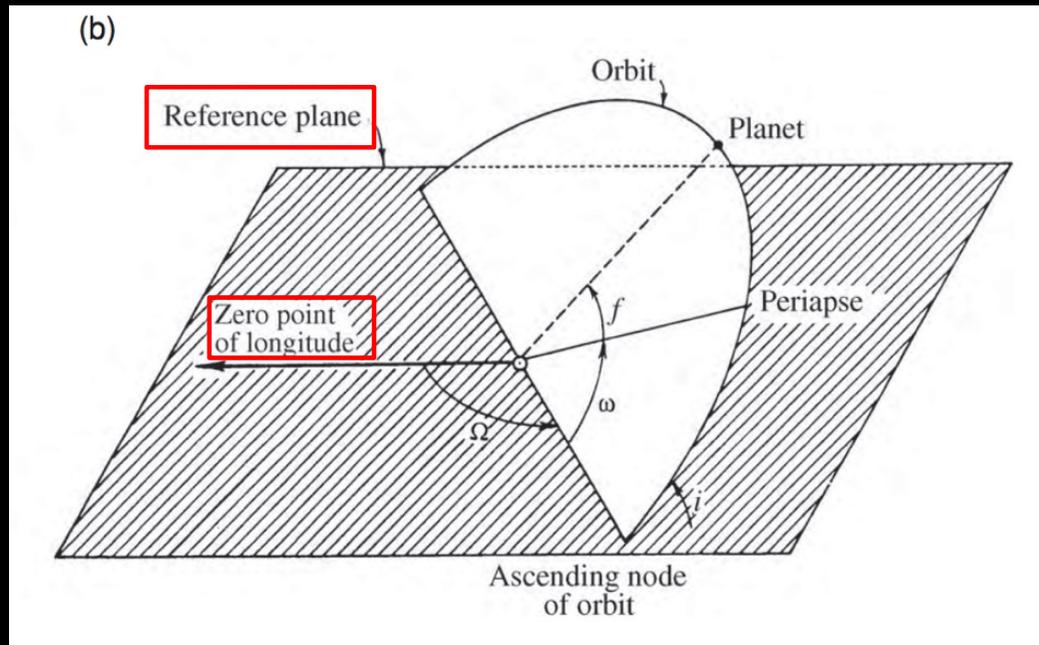
## Two Body Problem

- The relative motion of the two bodies is completely equivalent to that of a massless particle orbiting a *fixed* central mass  $M$ .



# Geometry of an Elliptical Orbit in 3D

Need one reference plane and a reference direction

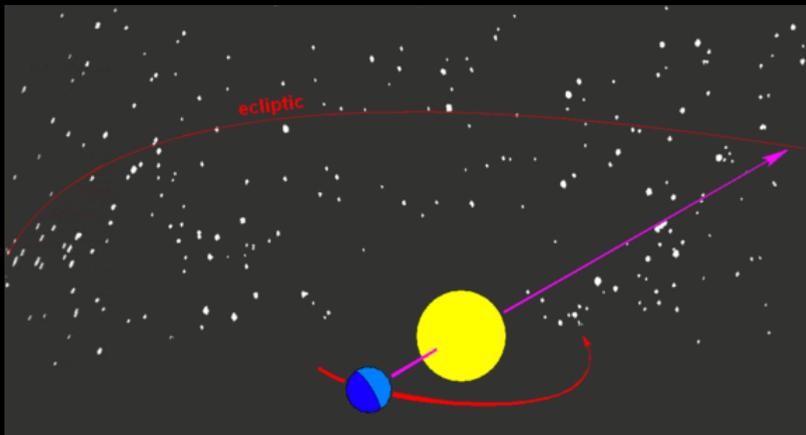


- $i$ : inclination (0-90: prograde; 90-180: retrograde)
- Line of nodes (two of them)
- $\Omega$ : longitude of the ascending node
- $\omega$ : argument of periaapse
- $f$ : true anomaly

# Geometry of an Elliptical Orbit in 3D

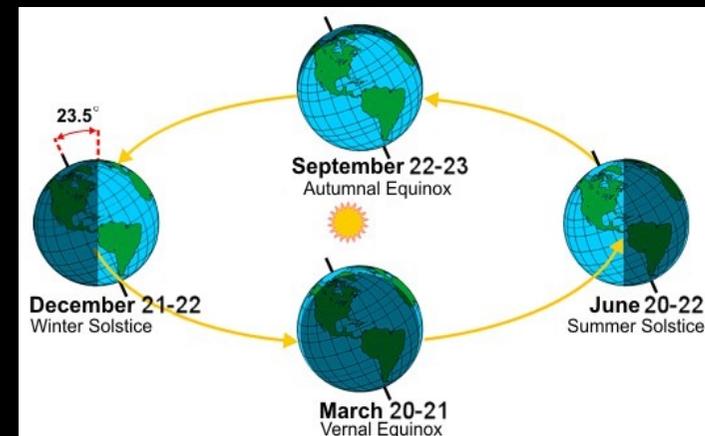
*For heliocentric orbits, the convention is to use the ecliptic plane (the orbital plane of Earth around the Sun) as the reference plane, and the vernal equinox as the reference direction*

## Ecliptic

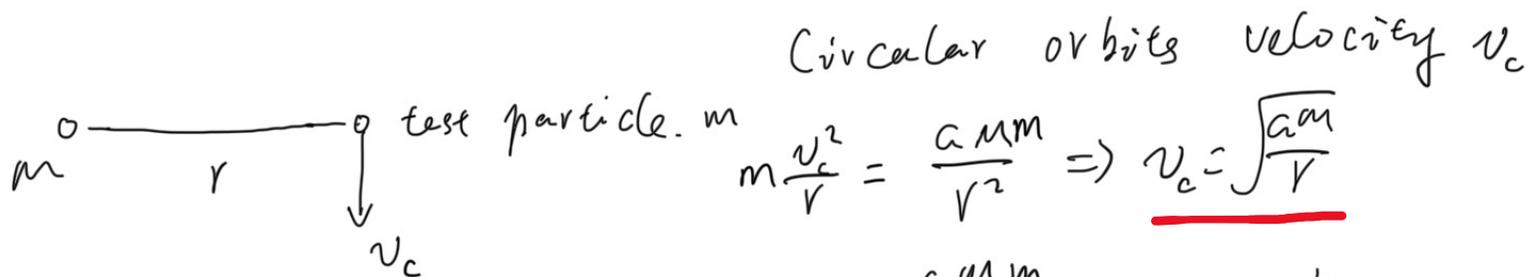


The Earth's orbital plane about the Sun

## Equinox



The instant of time when the plane of Earth's equator passes through the center of the Sun.



$$m \frac{v_c^2}{r} = \frac{GMm}{r^2} \Rightarrow \underline{v_c = \sqrt{\frac{GM}{r}}}$$

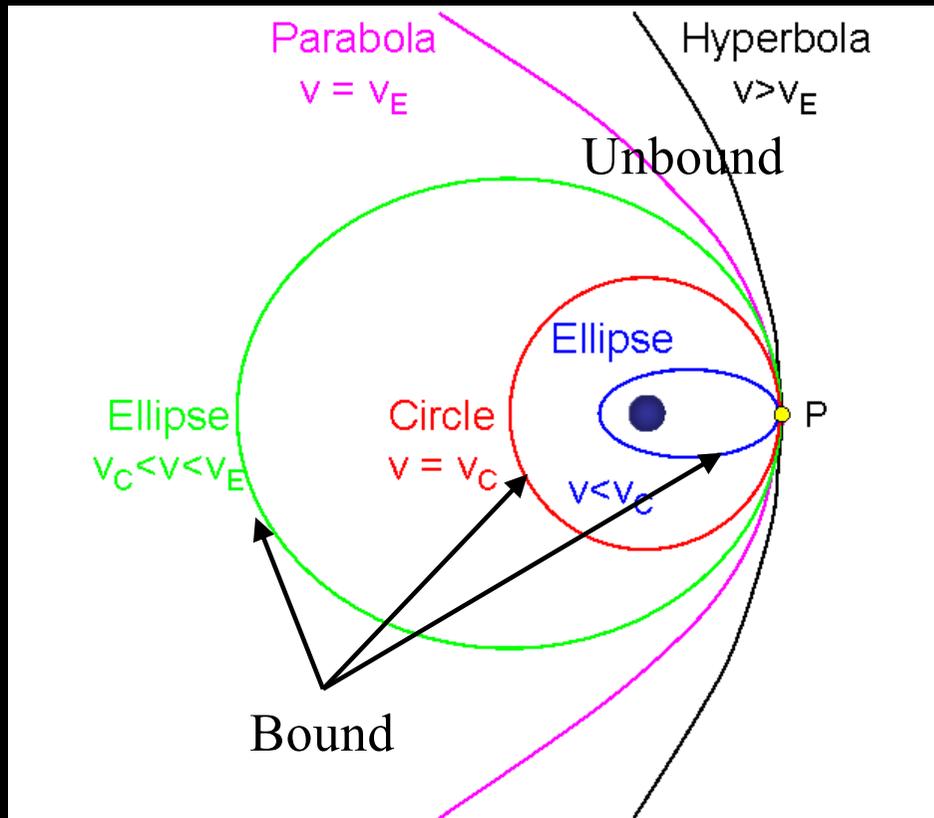
Total  $E$  of the particle  $E_{tot} = -\frac{GMm}{r} + \frac{1}{2} m v^2$

If  $E_{tot} < 0 \Rightarrow$  bound orbits

$\geq \Rightarrow$  unbound

$E_{tot} = 0 \Rightarrow \underline{v_e = \sqrt{\frac{2GM}{r}}}$  escape velocity

# Bound and Unbound Orbits

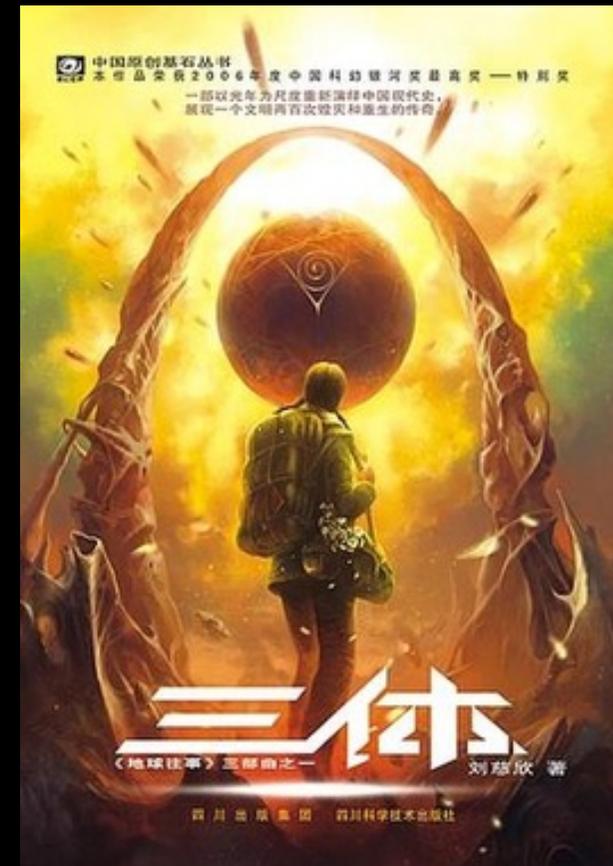
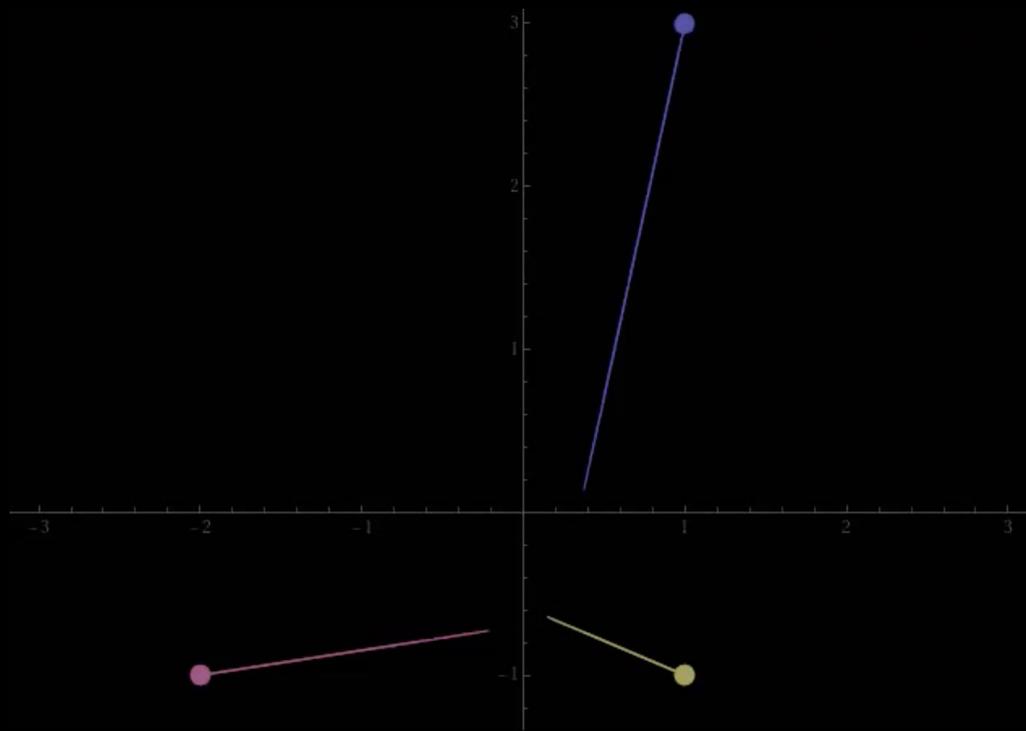


$$v_c = \sqrt{\frac{GM}{r}}$$

$$v_e = \sqrt{\frac{2GM}{r}} = \sqrt{2} v_c$$

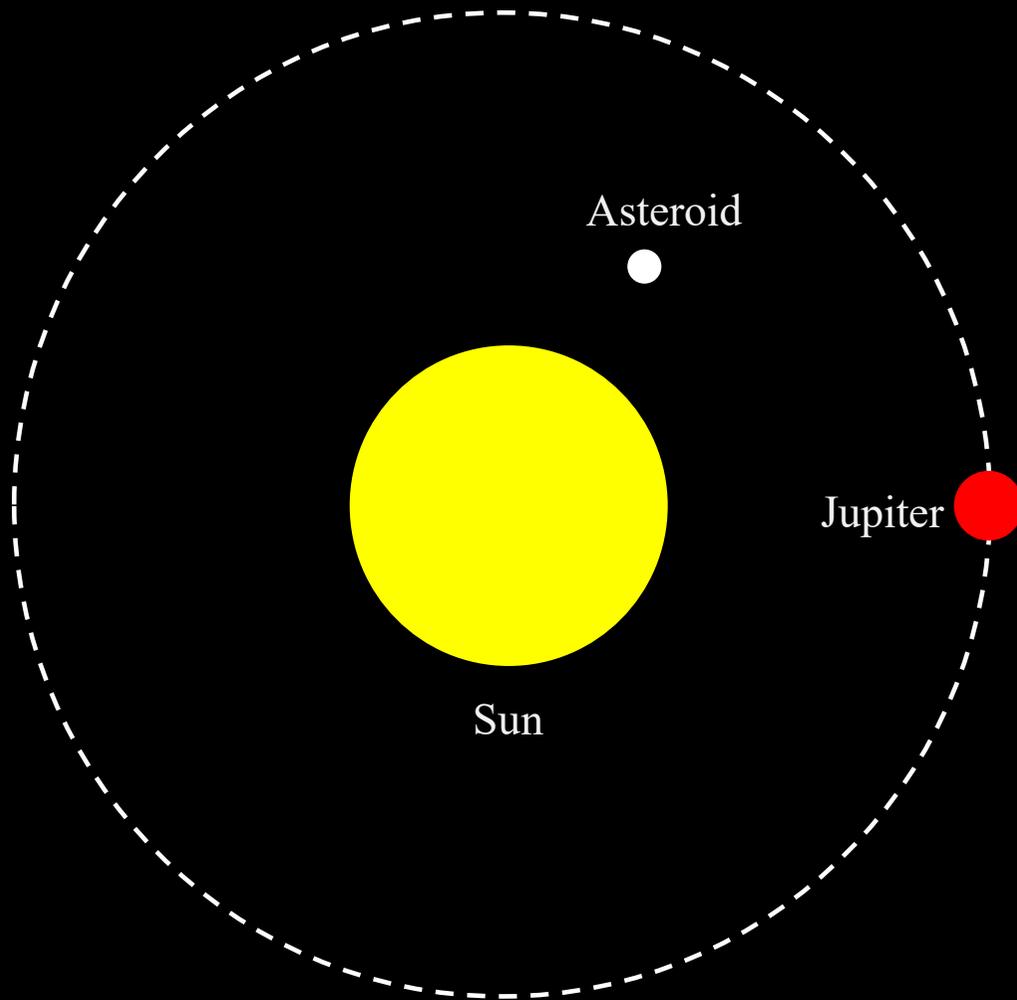
# The Three Body Problem (C.2.2)

Orbits are chaotic in the most general case



# The Three Body Problem (C.2.2)

Orbits are chaotic in the most general case



Ways to simplify the problem

- **Restricted three-body problem:** one of the bodies is of negligible mass
  - The 1<sup>st</sup> and 2<sup>nd</sup> bodies are on a stable orbit not affected by the 3<sup>rd</sup> body.
- **Circular restricted three-body problem:** the relative motion of the two massive particles is a circle
- **Planar circular restricted three-body problem:** all three bodies travel within the same plane.

# Circular Restricted Three-body Problem

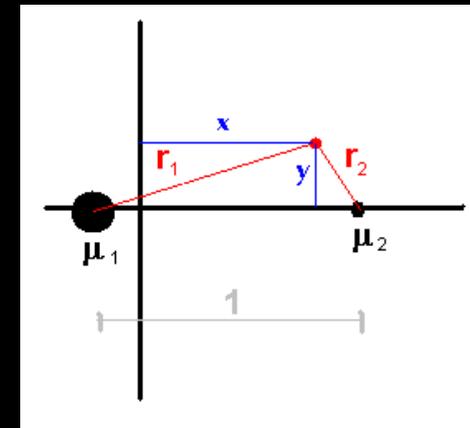
The relative motion of the two massive particles is a circle

- **Constant of motion**: a quantity that is conserved throughout the motion.  
Example: linear momentum of an isolated system
- **Jacobi's Constant** is a constant of motion in the circular restricted three-body problem (2.27)

$$C_J = 2U - v^2$$

$$U \equiv \frac{\Omega^2}{2} (x^2 + y^2) + \frac{Gm_1}{r_1} + \frac{Gm_2}{r_2}$$

- A given value specifies the magnitude of the test particle's **velocity** (in the rotating frame) as a function of **position**.
- Locations  $(x, y)$  with  $v^2 < 0$  is not permitted.
- The zero-velocity surface bounds the trajectory of a particle with fixed  $C_J$ .



Non-inertial frame rotates at the same rate as the binary

$$C_J = \underbrace{\int^2(x^2 + y^2) + \frac{2GM_1}{r_1} + \frac{2GM_2}{r_2}}_{\text{known}} \boxed{-v^2} = \underbrace{C_J}_{\text{const}}$$

$$v^2 = 0 \rightarrow 2U(x, y) = C_J \rightarrow \text{a curve } \underline{y(x)}$$

zero velocity curve

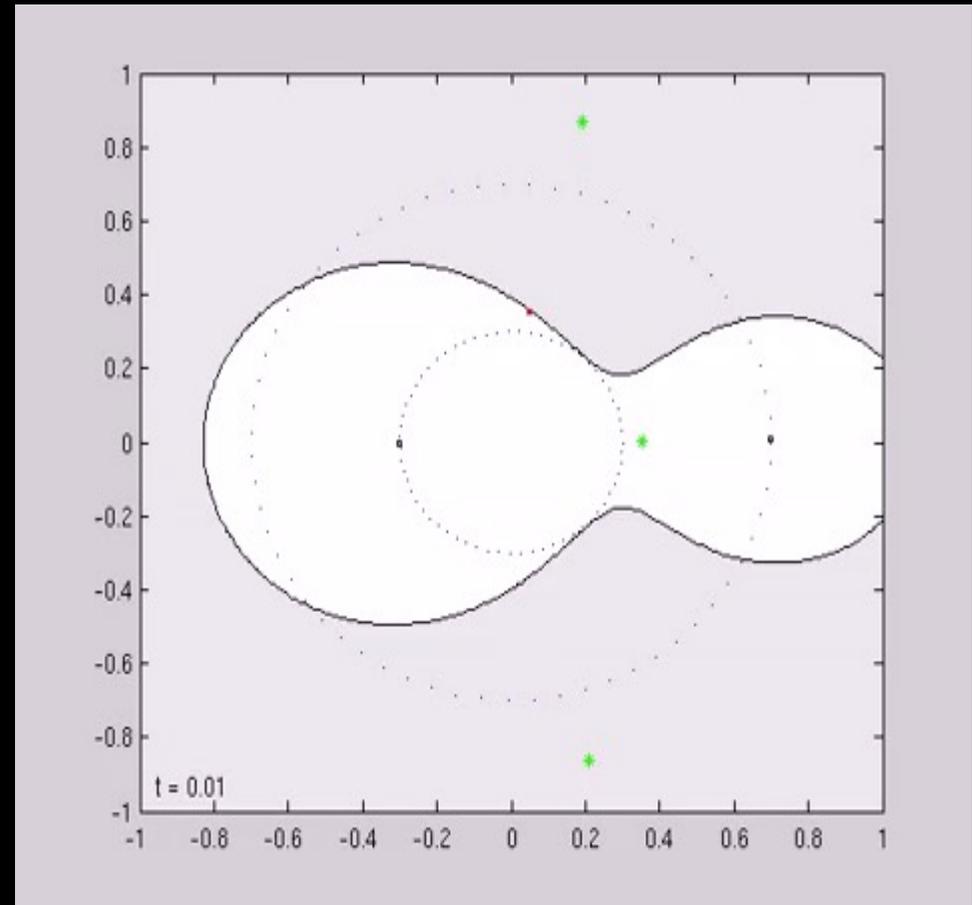
# Circular Restricted Three-body Problem

## Zero-velocity Curve

### Motion of a test particle

- Black dots: massive objects (mass ratio 7:3).
- Doted curves (circles): orbits of the two objects in inertial frame
- Solid curve: zero-velocity curve for a certain  $C_J$
- **White region: permitted**
- **Gray region: forbidden; the particle cannot enter these areas.**

*When the particle approaches the zero-velocity curve, its speed in the co-rotating frame approaches 0.*



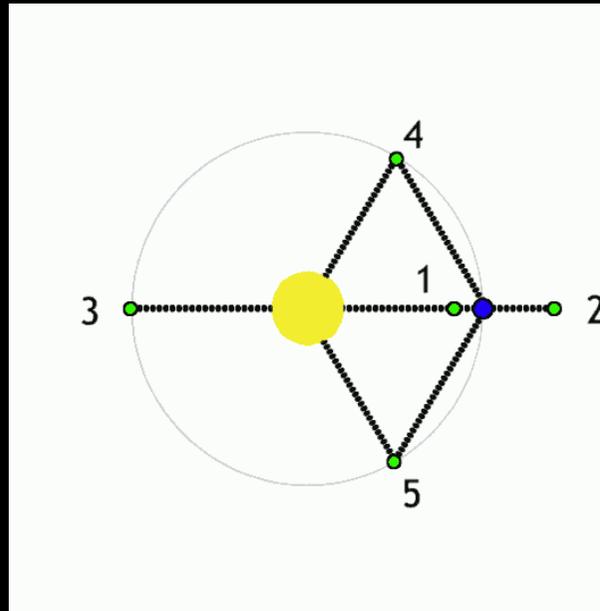
Justin Eldridge

<https://www.youtube.com/watch?v=xDLiUe8XYL0>

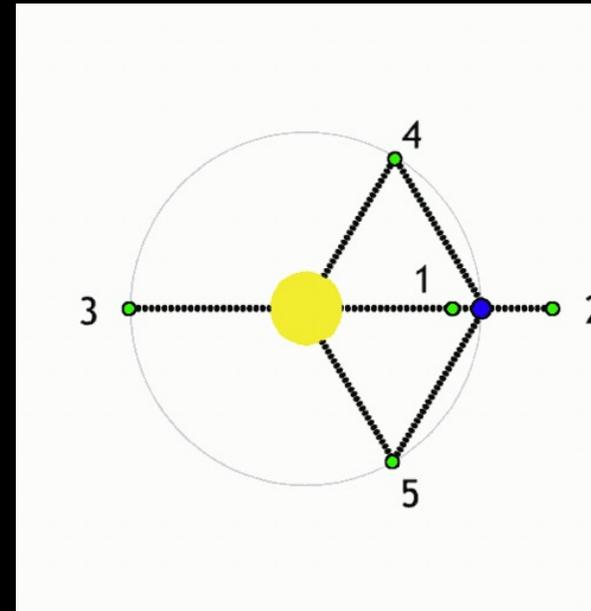
# Five Lagrangian Points in Circular Restricted Three-body Problem

**Definition:** Points near two large bodies in orbit where a smaller object will **maintain its position relative to the large orbiting bodies.**

Inertial frame



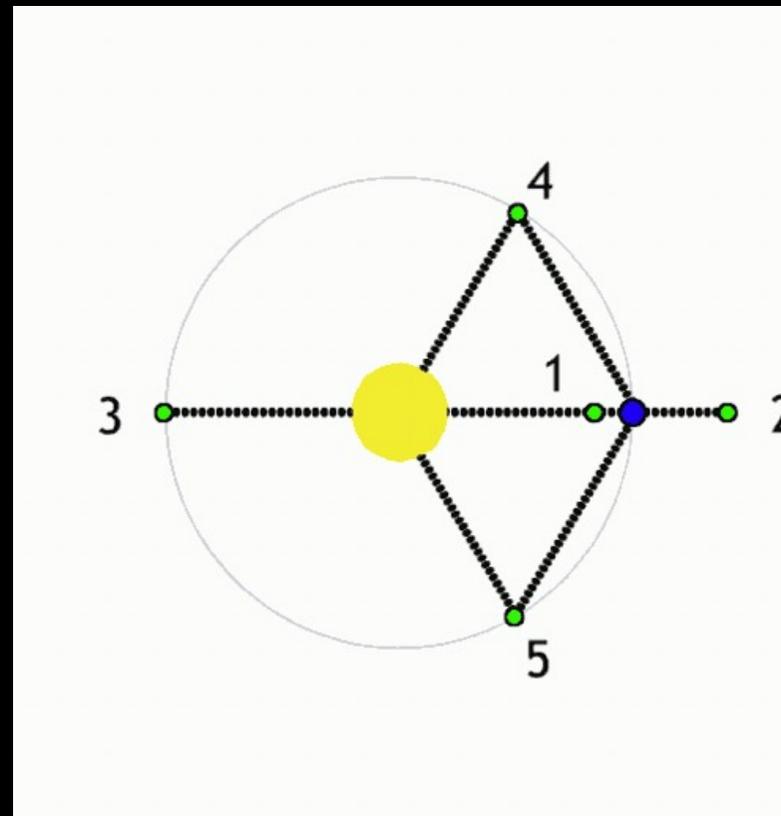
Co-rotating frame

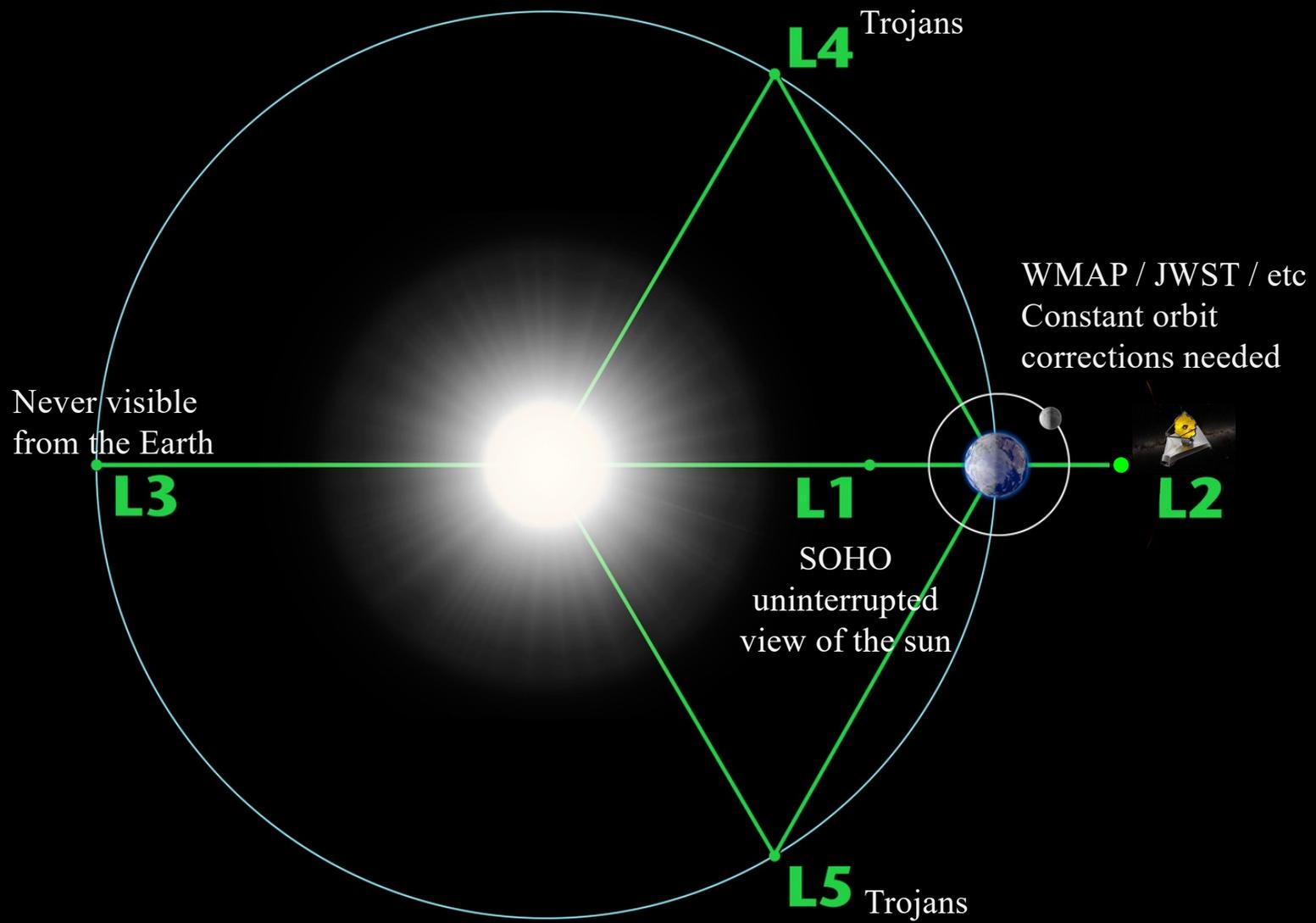


# Five Lagrangian Points

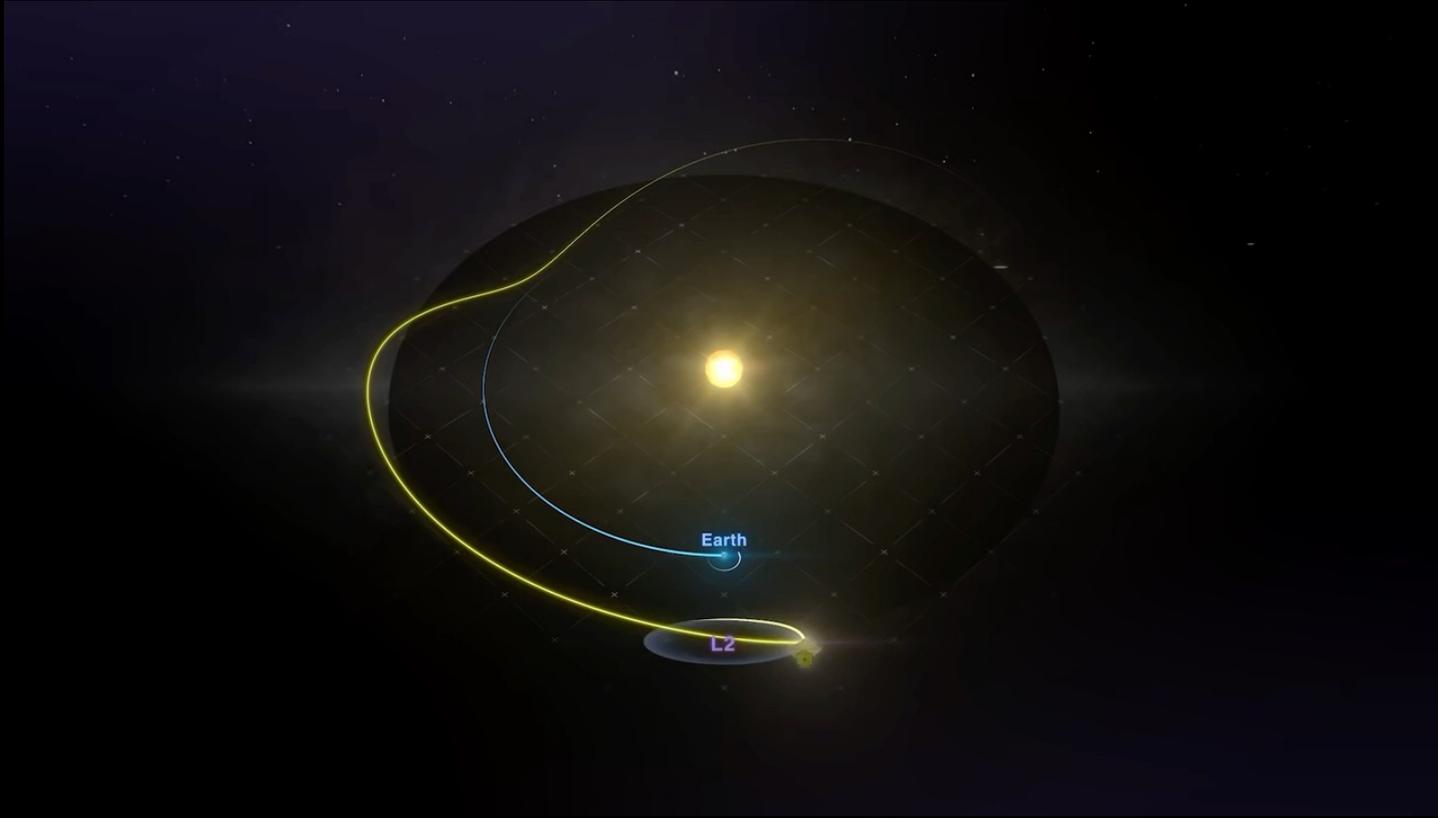
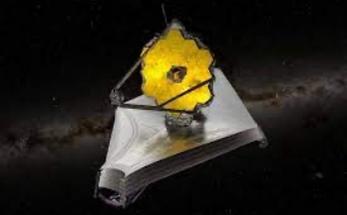
Co-rotating frame

- $L_1, L_2, L_3$ : along a line joining  $m_1$  and  $m_2$  (assignment)
- $L_4, L_5$ : equilateral triangles with  $m_1$  and  $m_2$  (assignment)
- All five points are in the orbital plane of the two massive bodies.
- Stability:
  - $L_1 / L_2 / L_3$ : unstable
  - $L_4 / L_5$ : stable for  $m_1/m_2 > 25$
  - Motions around  $L_4/L_5$ :  
Libration – oscillate back and forth



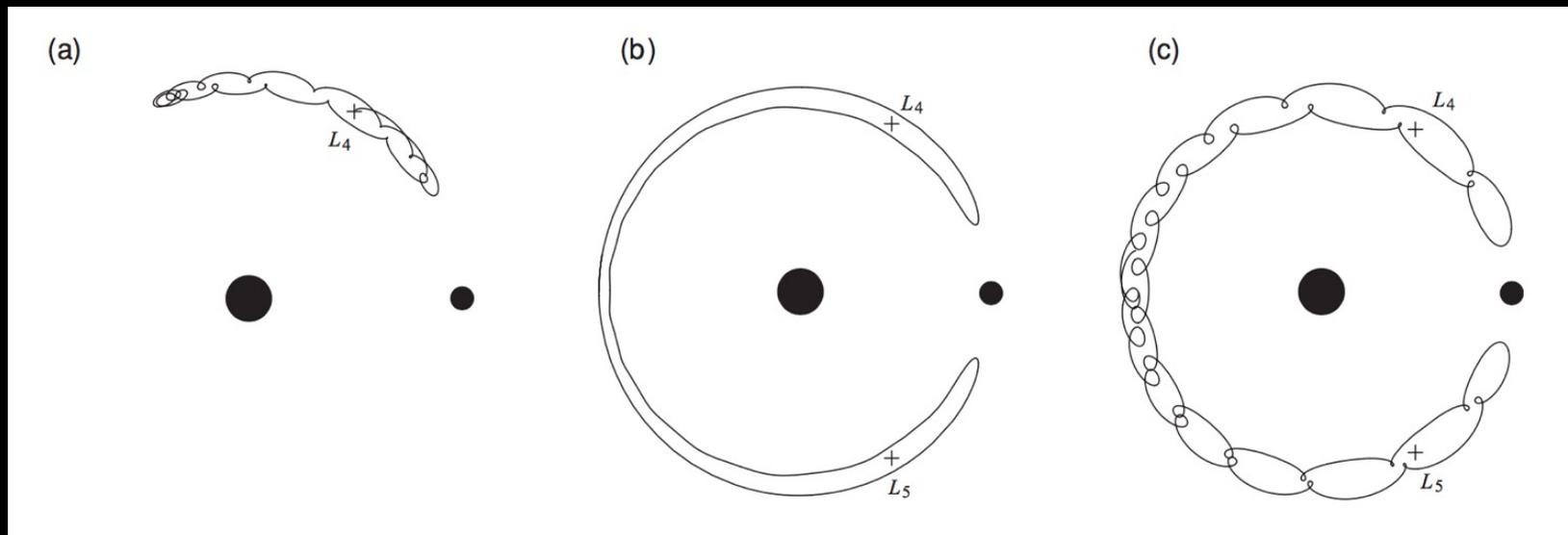


# The James Webb Space Telescope is Located at L2



# Orbits around L4 and L5 in the Rotating Frame

In a reference frame corotating with the planet



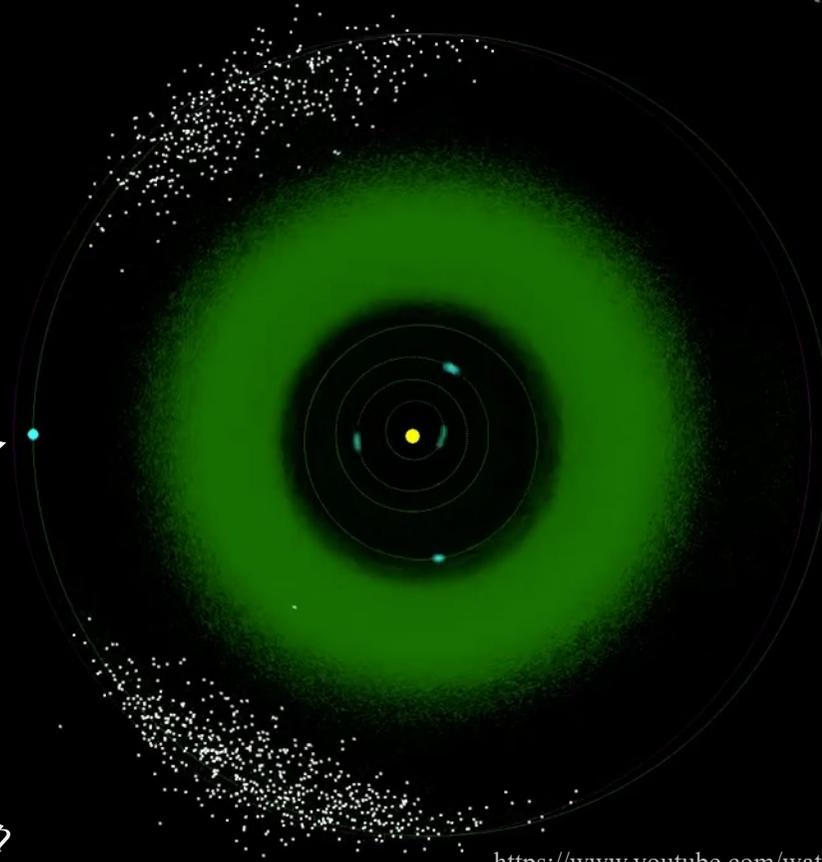
A tadpole orbit

A horseshoe orbit

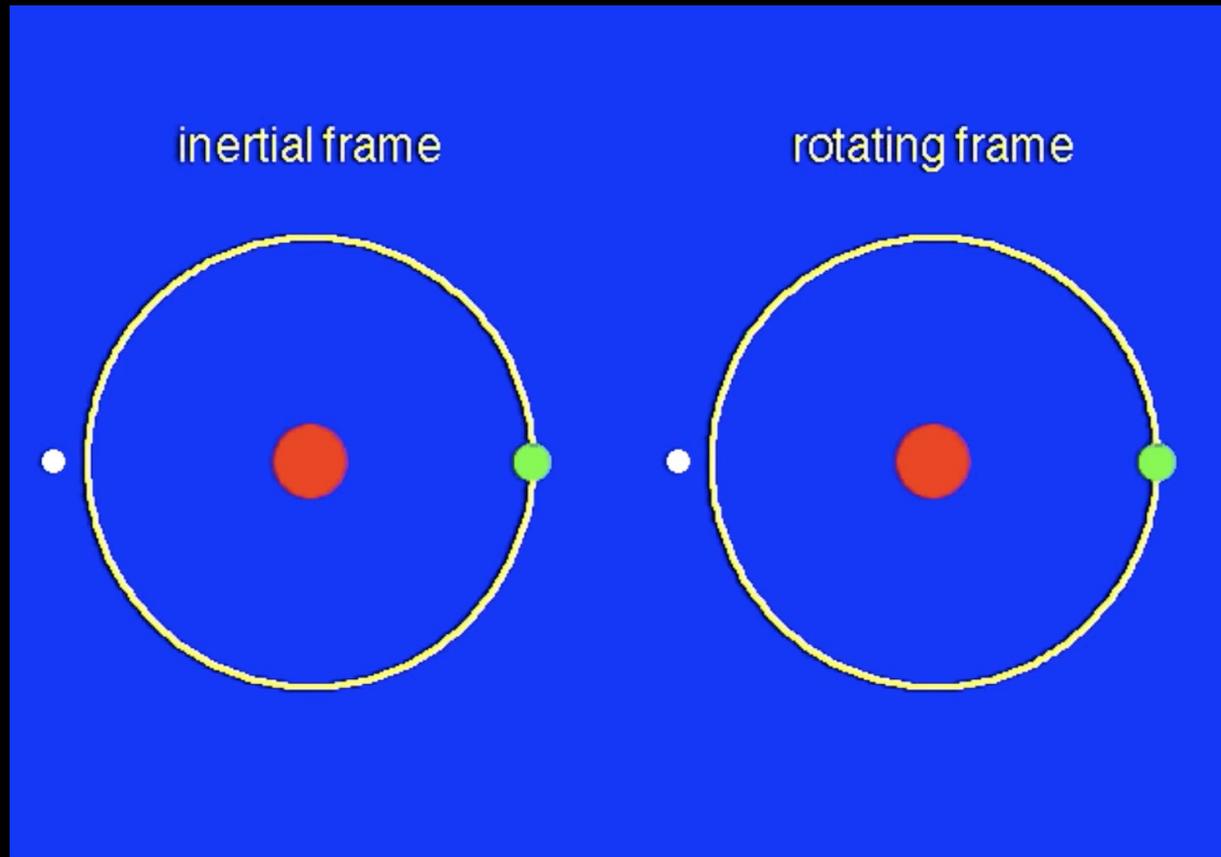
Another horseshoe orbit,  
with larger eccentricity

## Jupiter Trojan: Objects on Tadpole Orbits around L4 & L5

A frame corotating with Jupiter's mean motion  
(*but why Jupiter is still moving?*)



# Horseshoe orbit

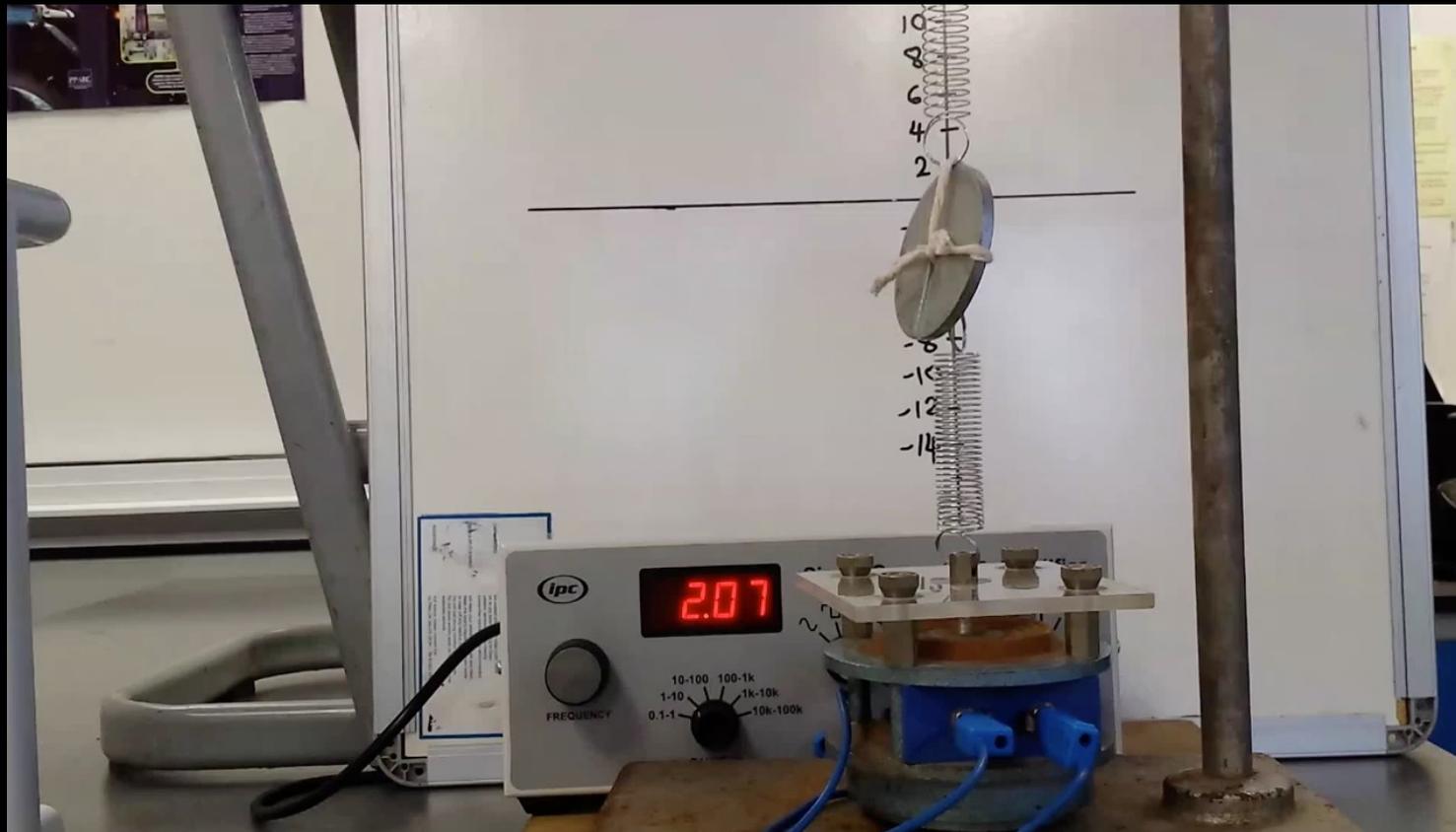


*Questions?*

*Assignment 1: derive Lagrangian points*

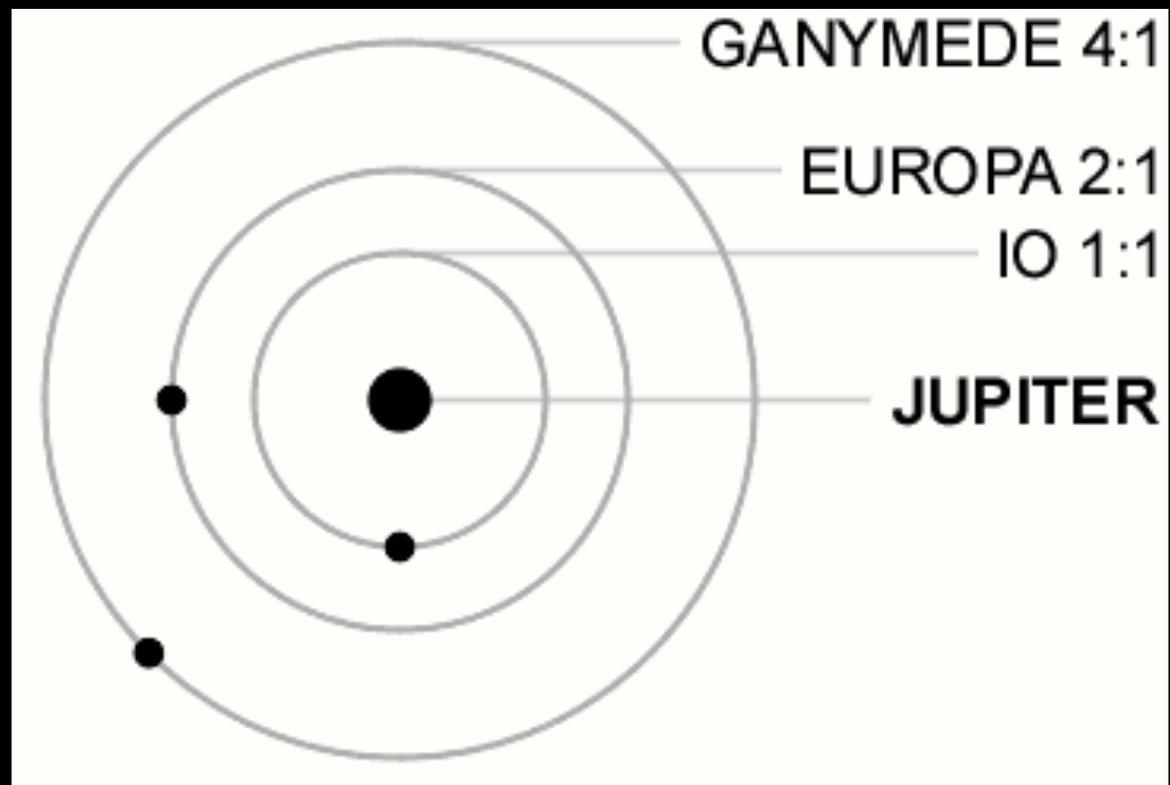
# Resonance (C.2.3)

when the frequency of an applied periodic force is equal or close to a natural frequency of the system

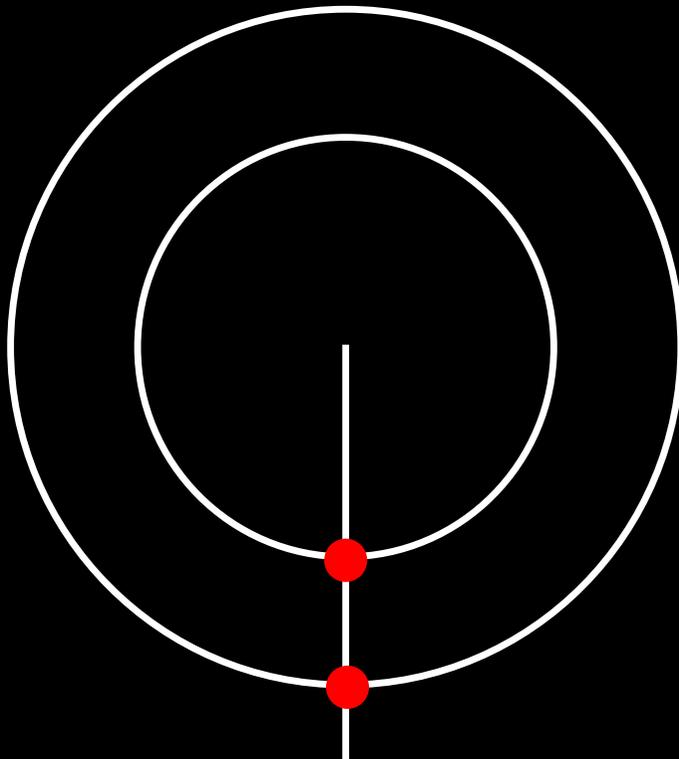


## Mean Motion Resonance

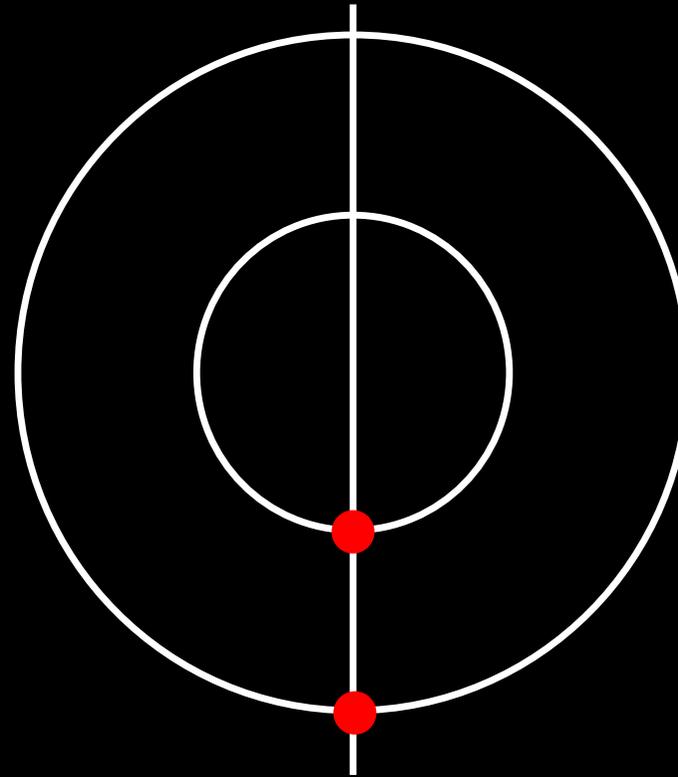
Orbital periods of two bodies are commensurate (e.g., have a ratio of the form  $N/(N+1)$ ,  $N/(N+2)$ , etc, where  $N$  is an integer).



1<sup>st</sup> Order MMR  
 $N / (N+1)$ , e.g., 2 / 3



2<sup>nd</sup> Order MMR  
 $N / (N+2)$ , e.g., 1 / 3

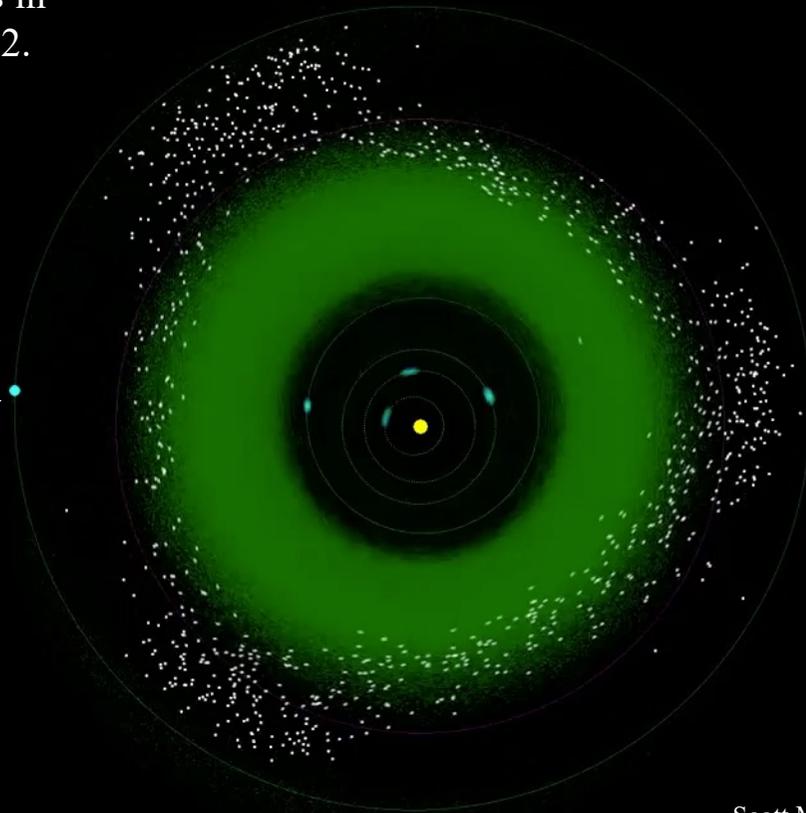


Conjunction: objects being at the same longitude in their orbits

# Objects in 3:2 Resonance with Jupiter

Objects in this group complete 3 orbits in the time that Jupiter takes to complete 2.

*A frame corotating with Jupiter's mean motion*



# Unstable Mean Motion Resonances with Jupiter in the Asteroid Belt

