

Assignment 3

1. Problem 9-8: Although in some respects Earth and Venus are ‘twin planets’, they have very different atmospheres. For example, the surface pressure on Venus is almost 2 orders of magnitude larger than that on Earth.
 - a. Calculate the mass of each atmosphere in unit of kilograms.
 - b. Recalculate these values for Earth, including Earth’s oceans as part of its ‘atmosphere’. (If all of the water above Earth’s crust were spread evenly over the planet, this global ocean would be ~3 km deep.)
 - c. Compare the values for the two planets and comment.

Hint: Appendix E may have atmospheric parameters of the two planets that you would need.

Solution: (a) Neglecting the small variations of g within each atmosphere because the thickness of the atmospheres is small comparing to the planetary radii, atmospheric mass can be calculated as:

$$M_{\text{atm}} = P \times 4\pi R^2 / g.$$

(Surface pressure multiplied by the area gives the gravitational force, divide by g to get mass.)

For Earth, using $P = 101.3\text{kPa}$ and $R_{\oplus} = 6370\text{ km}$, $M_{\text{atm}} = 5 \times 10^{18}\text{ kg}$. For Venus, $P = 9322\text{ kPa}$, $R = 6052\text{ km}$, so $M_{\text{atm}} = 5 \times 10^{20}\text{ kg}$.

(b) A 3 km layer of water has the mass: $4\pi R^2 \rho \Delta R = 1.4 \times 10^{21}\text{ kg}$.

(c) Without considering water, the atmosphere of Venus is 100 times as massive. Adding in the water content in the Earth’s oceans, Earth’s atmosphere + hydrosphere is about 3 times as massive as that of Venus.

2. Problem 6-15.

- a. What is the minimum impact velocity of a meteoroid when it enters the Earth’s atmosphere?
- b. In general, smaller meteoroid is slowed down more substantially. Determine the minimum radius of an iron meteoroid ($\rho = 8000\text{ kg m}^{-3}$) make it through Earth’s atmosphere without being substantially slowed down.

Hint: what is the condition to “substantially slowed down” a meteoroid? We talked about it in the class. You can also find the definition in S.6.4.3.

- c. Determine the minimum radius for a meteoroid of similar composition to make it through Venus’s atmosphere without being substantially slowed down.

Hint: Appendix E may have numbers you need in this calculation.

Solution: a) The minimum impact velocity is the escape velocity of the Earth, 11 km/s. If a meteoroid has a velocity smaller than this, then it cannot be originated from a large distance from the Earth. You can have the following thought experiment: what if you reverse the trajectory of the impactor – can it go anywhere?

b)

6.4.3 Impact Modification by Atmospheres

Hitherto we discussed impacts on airless bodies. If the target is enveloped by a dense atmosphere, as are Earth and Venus, impacts may be modified extensively. Projectiles can be completely vaporized while plunging through the atmosphere and never hit the ground. Or a projectile might break up into many pieces. Such fragmentation is thought to have led to the formation of **crater clusters** on Venus. No analogous close groupings are observed on the lunar surface. Atmospheric drag can slow down a small meteoroid so that it hits the surface at merely the terminal velocity (eq. 11.3). Larger bod-

The mass displaced during a meteor's fall through the atmosphere is $\sigma_\rho A / \sin \theta$, where σ_ρ is the mass of the atmosphere per unit area, A is the body's cross-sectional area and θ is the angle that the trajectory makes with the surface. The meteor is slowed substantially when it encounters an atmospheric mass about equal to its own mass. Significantly smaller bodies are slowed close to the terminal velocity before hitting the surface. It follows that the radius of a typical iron meteor must be larger than 1 m for it to hit Earth in a hypersonic impact (Problem 6-15). Note that when a meteor breaks up, A is increased, typically several fold.

However, actual meteors do not remain constant in size as they pass through the atmosphere. As their exteriors are heated to melting and even

A meteoroid is slowed substantially if it passes through its mass of gas. For Earth, the mass per unit area of the atmosphere is $\Sigma = p/g \sim 10^4 \text{ kg/m}^2$, where p is the atmospheric pressure (10^5 pa). An iron meteoroid smaller than ~ 1 meter in radius will be slowed appreciably.

c)

Table E.2 Terrestrial Planets: Geophysical Data

	Mercury	Venus	Earth	Mars
Mean radius R (km)	2440	6051.8	6371.0	3389.9
Mass ($\times 10^{24}$ kg)	0.3302	4.8685	5.9736	0.64185
Density (kg m^{-3})	5427	5204	5515	3933
Flattening ϵ			1/298.257	1/154.409
Semimajor axis			6378.136	3397
Sidereal rotation period	58.6462 d	-243.0185 d	23.934 19 h	24.622 962 h
Mean solar day (in days)	175.9421	116.7490	1	1.027 490 7
Polar gravity (m s^{-2})			9.832 186	3.758
Equatorial gravity (m s^{-2})	3.701	8.870	9.780 327	3.690
Core radius (km)	~ 1600	~ 3200	3485	~ 1700
Obliquity to orbit (deg)	~ 0.1	177.3	23.45	25.19
Sidereal orbit period (yr)	0.240 844 5	0.615 182 6	0.999 978 6	1.880 711 05
Escape velocity v_e (km s^{-1})	4.435	10.361	11.186	5.027
Geometric albedo	0.106	0.69	0.367	0.150

Table E.10 Atmospheric Parameters for Venus, Earth, Mars and Titan^a

Parameter	Venus	Earth	Mars	Titan
Mean heliocentric distance (AU)	0.723	1.000	1.524	9.543
Geometric albedo A_0	0.69	0.367	0.15	0.21
Bond albedo	0.77	0.306	0.25	0.20
Surface temperature (K)	737	288	215	93.7
Equilibrium temperature (K)	232	255	210	85
Exobase temperature (K)	270-320	800-1250	200-300	149
Surface pressure (bar)	92	1.013	0.00636	1.47
Scale height at surface (km)	16	8.5	11	20
Adiabatic lapse rate (K/km)	10.4	9.8	4.4	1.4

^a Venus albedos from <https://nssdc.gsfc.nasa.gov/planetary/factsheet/venusfact.html>. All other values from Table 4.2 in de Pater and Lissauer (2010). References are provided therein.

Venus' atmospheric pressure is about 100 times higher than that of the Earth. Thus, mass per unit area of the atmosphere is $\sim 10^6 \text{ kg/m}^2$ (the gravity of the two planets is similar). An iron meteoroid smaller than ~ 100 meters in radius is slowed appreciably.

3. Problem 9-4: Mercury's mean density is $\rho = 5430 \text{ kg/m}^3$. This value is very close to the planet's uncompressed density. If Mercury consists entirely of rock ($\rho_r = 3300 \text{ kg/m}^3$) and iron ($\rho_i = 7950 \text{ kg/m}^3$), calculate the planet's fractional abundance of iron by mass.

Solution: Mercury's mass $m = m_i + m_r$, where m_i is the mass of iron, and m_r is the mass of rock

$$\frac{m}{\frac{m_r}{\rho_r} + \frac{m_i}{\rho_i}} = \rho \Rightarrow \frac{m}{\frac{m - m_i}{\rho_r} + \frac{m_i}{\rho_i}} = \rho$$

$$\Rightarrow m = \rho \left(\frac{m - m_i}{\rho_r} + \frac{m_i}{\rho_i} \right) = \frac{\rho}{\rho_r} (m - m_i) + \frac{\rho}{\rho_i} m_i$$

$$\Rightarrow \frac{\rho}{\rho_r} \left(1 - \frac{m_i}{m} \right) + \frac{\rho}{\rho_i} \frac{m_i}{m} = 1 \Rightarrow \frac{\rho}{\rho_r} - \frac{\rho}{\rho_r} \frac{m_i}{m} + \frac{\rho}{\rho_i} \frac{m_i}{m} = 1$$

$$\Rightarrow \frac{\rho}{\rho_r} - 1 = \frac{m_i}{m} \left(\frac{\rho}{\rho_r} - \frac{\rho}{\rho_i} \right) \Rightarrow \frac{m_i}{m} = \frac{\frac{\rho}{\rho_r} - 1}{\frac{\rho}{\rho_r} - \frac{\rho}{\rho_i}} = 67\%$$

4. Problem 10-4

Solution:

a) Flux at Titan is $1 \times 10^{16} \div 9.5^2 = 1.1 \times 10^{14} \text{ photons/m}^2/\text{s}$. The global average flux is $1 \times 10^{16} \div 9.5^2 \div 4 = 2.77 \times 10^{13} \text{ photons/m}^2/\text{s}$.

b) H_2 escape flux = $2.77 \times 10^{13} / 2 = 1.38 \times 10^{13} \text{ m}^2/\text{s}$

C_2H_6 creation rate = $1.38 \times 10^{13} \text{ m}^2/\text{s}$

CH_4 destruction rate = $2.77 \times 10^{13} \text{ m}^2/\text{s}$

c) The number of moles of gas in a column of Titan's atmosphere is

$N = p / m_{\text{mu}} / g = 1.5 \times 10^5 \div 0.028 \div 1.4 = 3.83 \times 10^6 \text{ moles/m}^2$.

The column density (the number of molecules in a column) of methane is $0.05 \times 3.83 \times 10^6 \times 6 \times 10^{23} = 1.15 \times 10^{29} \text{ CH}_4 \text{ molecules/m}^2$.

The lifetime of the current reservoir of CH_4 in Titan's atmosphere is roughly $1.15 \times 10^{29} \div (2.77 \times 10^{13}) = 4.14 \times 10^{15} \text{ sec} = 130 \text{ million years}$.

d) $1.15 \times 10^{29} \div 2 \div (6 \times 10^{23}) = 9.6 \times 10^4 \text{ moles/m}^2$.

e) $9.6 \times 10^4 / (130 \text{ million years}) \times (4.5 \text{ billion years}) \times 0.030 \text{ (kg/mole)} / 550 = 180 \text{ m}$

f) CH_4 : (bp) 111 K; (fp) 90.7 K

C_2H_6 : (bp) 184.6 K; (fp) 90.4 K.

g) Titan's surface temperature is 93.7K (Table E.10), thus CH_4 and C_2H_6 are both in liquid form (their fp increases ever so slightly with pressure). Cassini-Huygens scientists expected to land in a liquid methane-ethane "ocean" 100 m deep, because photochemical decomposition of methane over 4.56 billion yrs would have left 100 m of liquid ethane at the surface. Imaging in

infrared wavelengths showed features that resembled rivers, lakes, oceans, etc, but they actually found land, probably because not all the radar-dark features that resembled lakes or seas are actually liquid bodies; smooth surfaces could also look similar. There are actually hydrocarbon liquid lakes on Titan, mostly at high latitudes. Also, there may not be that much CH₄ to last for 4.5 billion, otherwise we'll be catching a very special time in Titan's history.

5. Problem 11-8: Calculate the fractional abundance of ²³⁴U in naturally occurring uranium ore.

Hint 1: Use the first decay chain shown in Figure 11.17.

Hint 2: ²³⁵U can be ignored.

Solution: The half-life of ²³⁴U (2.47×10^5 years) is much shorter than the age of the Earth (≈ 4.55 Gyr). Thus, any primordial ²³⁴U must have been completely exhausted long time ago, and any ²³⁴U present now must be in dynamic equilibrium with ²³⁸U, i.e., in every second the amount of ²³⁴U generated by ²³⁸U decay must equal to the amount of ²³⁴U consumed by ²³⁴U decay. Also, ²³⁴Th and ²³⁴Pa have half-life too short and can be ignored.

$$\frac{dN_{234}}{dt} = \underbrace{\frac{dN_{234}}{dt}}_{\substack{N_{238} \\ t_m(238 \rightarrow 234)} \text{ production}} + \underbrace{\frac{dN_{234}}{dt}}_{\substack{N_{234} \\ t_m(234 \rightarrow xxx)} \text{ consumption}} = 0$$

$$\Rightarrow \frac{N_{238}}{4.51 \times 10^9 \text{ yr}} = \frac{N_{234}}{2.47 \times 10^5 \text{ yr}} \Rightarrow \frac{N_{234}}{N_{234} + N_{238}} = \frac{N_{234}}{N_{\text{total}}} = 0.0055\%$$

(note that $4.51e9$ in the last row should be 4.47)

6. Problem 12-3. Suppose a comet has a velocity of 40 km/s at perihelion. The perihelion distance is 1 AU. Calculate the aphelion distance, the velocity of the comet at aphelion and the orbital period of the comet.

Hint: The orbital parameters of an object can be determined if we know the heliocentric distance and velocity of the object at any one position on its orbit. S.2.1.6 shows you how that can be done.

Solution:

$$v_c = \sqrt{\frac{GM}{r}} \quad (2.21)$$

The total energy of the system, E , is a conserved quantity:

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a}, \quad (2.22)$$

where the first term in the middle expression is the kinetic energy of the system and the second term is potential energy. For circular orbits, the second

Using eq. (2.22), the energy of the comet per unit mass is given by:

$$\frac{E}{m} = -\frac{GM_{\odot}}{2a} = \frac{v^2}{2} - \frac{GM_{\odot}}{r}$$

Solve this equation, we get the semi-major axis a

$$a = \left(\frac{2}{r} - \frac{v^2}{GM_{\odot}} \right)^{-1} = \left(\frac{2}{1.496 \times 10^{11}} - \frac{40,000^2}{6.674 \times 10^{-11} \times 1.989 \times 10^{30}} \right)^{-1} = 7.6 \times 10^{11},$$

which is 5.08 AU. The aphelion distance is thus $[2a - \text{perihelion distance}] = 9.16$ AU. The velocity of the comet at the aphelion can be calculated using angular momentum conservation, $v_{\text{perihelion}} \times r_{\text{perihelion}} = v_{\text{aphelion}} \times r_{\text{aphelion}}$, thus $v_{\text{aphelion}} = 40/9.16 = 4.4$ km/s. The period can be calculated using Kepler's third law, which gives $P = 11.4$ years.

7. Problem 12-4(a). Show that if the exponent $\zeta = 4$ in equation (12.1a), then the mass is divided equally among equal logarithmic intervals in radius.

Solution:

(a)

$$N(R) = N_o \left(\frac{R}{R_o} \right)^{-\zeta}$$

$$M(R) = N(R) \frac{4\pi\rho}{3} R^3$$

pick an arbitrary value of R_1 . For $\zeta = 4$, we have

$$M\left(\frac{R_1}{2} < R < R_1\right) = \int_{\frac{R_1}{2}}^{R_1} N_o \frac{4\pi\rho}{3} r^{-1} R_o dr = \frac{4\pi\rho}{3} N_o R_o^4 \ln 2$$

independently of R_1 , so mass is shared equally among logarithmic intervals in radius.

8. Problem 13.3: Calculate the Roche limit of Jupiter and Saturn assuming a satellite density of 1 g/cm^3 , appropriate for dirty snowballs. Compare your results with the observed ring sizes and then comment, e.g., what could you possibly infer about the origin of these rings? You could assume the rings have not moved much radially since their formation.

Solution: The Roche limit is given by equation (13.8). Plug in numbers, we get

$$a_{R,\text{Jup}} = 1.9 \times 10^5 \text{ km}$$

$$a_{R,\text{Sat}} = 1.3 \times 10^5 \text{ km}$$

Jupiter's rings span from half the Roche limit value calculated using the ice ball approximation to nearly that value. Saturn's main rings extend to just beyond the Roche limit for ice-ball bodies, while the G-ring and the faint E-ring extend well beyond it. These implies that Jupiter's rings and Saturn's main rings could possibly be produced as moons (or other Solar system minor objects) in the past approached the planets then get tidally disrupted to become rings. Meanwhile, Saturn's G and E rings are unlikely produced by this mechanism.

Short questions:

What is the atmosphere of Mars mainly made of?

CO₂

What is the atmosphere of Venus mainly made of?

CO₂

What is the atmosphere of Titan mainly made of?

N₂

What are the main compositions of the atmosphere of Jupiter and Saturn?

H₂ and He

What is the shape of a rotating planet that is in hydrostatic equilibrium such as Jupiter?

Oblate

When did Late heavy bombardment occur?

About 700 Myr since the birth of the solar system (or 3.9 billion years ago)

What is the technique that is used the most (and often the only available) to assign an age to a body's surface?

Crater dating

What are the main compositions of the solar wind?

electrons, protons, alpha particles

Why magnetic fields around planets cannot be caused by permanent magnetism in a planet's interior?

Curie point for iron is near 800 K; at higher temperatures, iron loses its magnetism.

A permanent magnet would gradually decay away over the characteristic Ohmic dissipation time, and for the Earth the timescale is 10^4 — 10^5 years.

Earth's magnetic field has reversed its direction frequently during geologically recent times; a permanent magnet would not behave in this manner.

Which moon in the solar system has the thickest atmosphere?

Titan

How many moons with a size bigger than 1 km does Mars have?

2

What's the largest moon of Saturn?

Titan

Which object is the most volcanically active in our Solar System?

Io (Jupiter's moon)

Why Europa is considered as a top destination in search for extraterrestrial life in the solar system?

Possible subsurface ocean

Composition-wise, meteorites can be classified into three kinds. What are they?

Stony, Iron, Stony-Iron

And which one is the most abundant?

Stony

Apart from the 8 planets and their moons, where are other major objects located in the solar system?

Asteroid belt, Kuiper belt, Oort Cloud

Where is the asteroid belt located?

Between Mars and Jupiter

Where is the Kuiper belt located?

Beyond the orbit of Neptune, roughly 30-50 AU

Comets can typically have two tails. What are they made of?

A plasma tail (ionized gas), and a dust tail

Out of the 4 giant planets in the solar system, how many have rings?

All of them

Members in an asteroid family have what in common?

Origin

What is the primary reservoir of short-period comets?

Kuiper Belt

What do most long period and dynamically new comets come from?

Oort Cloud

Which planet has the most massive rings in the solar system?

Saturn