Assignment 1

Problem 1

Scale the entire solar system such that the diameter of the Earth is 1 cm.

1. Calculate the size of the Sun and the other planets.
2. Calculate the distances from the planets to the Sun.
3. How far is the nearest star to the Sun in this system?

Problem 2

Lagrangian points

a. L1, L2, and L3 are along the line joining two masses m_1 and m_2 in the circular restricted three body problem. Assuming R is the distance between the two main objects, and $m_2 << m_1$, find out the separation between L1 and m_2.
   Hint 1: start from the definition of L points.
   Hint 2: when $m_2 << m_1$, L1 and L2 are very close to m_2.
b. Find out the separation between L2 and m_2.
c. Evaluate L1 and L2 locations in the case of the Sun-Earth system. Express the results in units of both km and in Earth-Moon separation.
d. Is L3 closer or further away from the COM than m_2? Why?
e. L4 (and L5) forms an equilateral triangle with m_1 and m_2. Prove that it is a Lagrangian Point in the special case of $m_2 = m_1$.

Problem 3

What's the time interval between two consecutive tides induced by the moon?

Hint 1: how many tides induced by the moon between two successive moonrises?
Hint 2: You might want to think about time in sidereal time. If you are unfamiliar with the concept, Wikipedia and Figure 2.21 in the book might be helpful.

Problem 4

The sun is losing 6×10^{12} grams of mass every second at the moment via its solar wind and by converting mass into radiation. The Earth orbits the Sun. As the mass of the Sun decreases, the Earth is held a bit less strongly, and its orbit expands.

a. Derive Eqn. (2.65) in the textbook. You can assume circular orbits.
b. Evaluate the expansion rate in units of cm/yr.

Hint: the orbital angular momentum of the Earth is conserved in this process.

Problem 5

A small asteroid on a circular orbit around the Sun releases a dust grain. Assuming at the moment of release the dust grain achieves the same orbital velocity around the Sun as the asteroid, and the mass of the asteroid is too small to affect the dynamics of the grain. What's the minimum β value (equation 2.60) that the dust grain needs to have in order to escape the solar system? Ignore all other bodies in the Solar system (except the Sun of course). Please derive your answer instead of directly giving a number.

Problem 6

Prove the virial theorem in the case of m_1 and m_2 orbiting each other in circular orbits in the general case, i.e., without assuming $m_1 >> m_2$. The two objects are separated by a distance r.

1. What is the total kinetic energy of the system?
2. What is the total gravitational potential energy?
3. Show that the virial theorem holds.