Assignment 4

Problem 1

Calculate the maximum radial velocity of the Sun to an alien observer due to the presence of the following planet on its current orbit (you can assume the planet is the only planet in the solar system in each calculation, and the planet is on a circular orbit).
Hint: what is the observing geometry for an alien observer to observe the maximum radial velocity of the Sun?

- Jupiter
- Saturn
- Earth
- Neptune

Solution:

- Jupiter: 12.4 m/s
- Saturn: 2.75 m/s
- Earth: 0.089 m/s
- Neptune: 0.28 m/s

Problem 2

14-3. **(a)** Calculate the probability of transits of the planets Venus and Jupiter being observable from another (randomly positioned) planetary system.

Solution:

$$P_{tr} = (R_\star + R_p)/a(1-e^2) \qquad (14.5)$$

$$R_\star = 6.95 \times 10^8 \text{m}.$$

For Jupiter: $R_J = 7.15 \times 10^7$ m; $a_J = 778.57 \times 10^6$ km; $e_J = 0.0484$; $P_J =$ 0.0987%.

For Venus: $R_V = 6.05 \times 10^6$ m; $a_V = 108.21 \times 10^6$ km; $e_V = 0.0067$; $P_V =$ 0.64%

Problem 3

What is the radius of a 1 Earth mass planet with (from inside out) 1/3 of its mass in Fe core, 1/3 in silicate mantle, and 1/3 in water (H_2O) outer region?

Solution: Read from Fig. 14.22: roughly 7500 km.

Problem 4

In typical microlensing exoplanet detections, the source star is in the galactic bulge at 8 kpc away, and the lens star is at 4 kpc away from us. Typically, how close do the source and lens stars need to be on the sky in order to trigger a microlensing event? You can assume a lens star mass of 1 solar mass.

Solution: Typically, the two stars need to get together within the Einstein radius of the lens star for a microlensing event to occur. Given the parameters, the Einstein radius is about 1 mas, or 4.8×10^{-9} radian.

Problem 5

14-8. (a) Calculate the ratio of the light reflected by Earth at 0.5 μm to that emitted by the Sun at the same wavelength.

(b) Calculate the ratio of the thermal radiation emitted by Earth at 20 μm to that emitted by the Sun at the same wavelength.

(c) Repeat the above calculations for Jupiter.

Solution:

(a) The fraction of the Sun's radiation that is reflected by Earth equals the cross-sectional area of the Earth multiplied by Earth's albedo and divided by the area of a sphere of radius equal to Earth's orbit:

$$\frac{\pi R_\oplus^2}{4\pi r_\oplus^2} \times \mathcal{A}_\oplus = \frac{0.3}{4} \times (\frac{6370}{1.5 \times 10^8})^2 \approx 1.3 \times 10^{-10}.$$

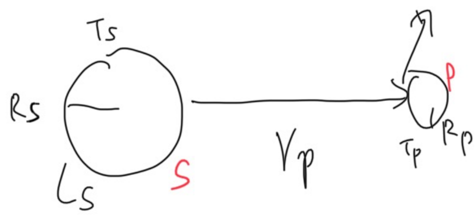
(b) The fraction of the Sun's total energy emitted by Earth in the thermal infrared is

$$\frac{4\pi R_\oplus^2 \sigma T_{\mathrm{eff},\oplus}^4}{4\pi R_\odot^2 \sigma T_{\mathrm{eff},\odot}^4},$$

which can be multiplied by the fraction of the Sun's luminosity that is emitted in this wavelength range.

Alternatively, take the ratio of surface areas of the bodies and multiply by the ratios of temperatures (Rayleigh-Jeans law, eq. 4.4): $R_\oplus^2/R_\odot^2(T_\oplus/T_\odot)$ $10^{-4} \times 4 \times 10^{-2} = 4 \times 10^{-6}$.

(c) The fraction of the Sun's radiation reflected by Jupiter $\approx 3 \times 10^{-9}$. Ratio of Jupiter's luminosity to that of the Sun in thermal IR $\approx 2 \times 10^{-4}$.



reflected light cross section

$$L_p = \underbrace{\frac{L_s}{4\pi r_p^2}}_{\text{flux at planet location}} \times \pi R_p^2 \times A_p$$

Albedo (angle dependent)

Brightness ratio at a given wave length. λ $\frac{L_s}{L_p}\Big|_\lambda$

$$\frac{L_p}{L_s} = \frac{\pi R_p^2}{4\pi r_p^2} A_p$$

$R_p = 5\times10^{-5}$ AU
$r_p = 1$ AU

$$= \frac{(5\times10^{-5})^2}{4\times1^2} A_p = 6\times10^{-10} A_p \sim 10^{-10}$$

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thermal emission (blackbody)

$$L_{s,\lambda} = \underbrace{4\pi R_s^2}_{\text{total surface area}} \times \underbrace{B_\lambda(T_s)}_{\text{planck F.}} \times \pi$$

$$L_{p,\lambda} = 4\pi R_p^2 \times B_\lambda(T_p) \times \pi$$

Blackbody radiation peaks at λ_{peak} (Wien's displacement law)

$$\lambda_{peak} = \frac{b}{T} \quad (\lambda_{peak,s} < \lambda_{peak,p})$$

If we work at $\lambda \gg \lambda_{peak,p}$, $B_\lambda(T) \sim \frac{2ck_BT}{\lambda^4}$, Rayleigh Jeans

$$\frac{L_{p,\lambda}}{L_{s,\lambda}} = \frac{4\pi R_p^2 B_\lambda(T_p)}{4\pi R_s^2 B_\lambda(T_s)} = \left(\frac{R_p}{R_s}\right)^2 \frac{T_p}{T_s}$$

Problem 6

14-14. Consider a terrestrial planet with Earth's radius orbiting 0.03 AU away from a cool M dwarf star with luminosity $\mathcal{L} = 10^{-3}\mathcal{L}_\odot$.

(a) Calculate the stellar flux intercepted by this planet (in J m^{-2}). What is this flux in units of the solar flux intercepted by the Earth?

(b) Planets orbiting this close to their star are likely to become 'tidally locked' and keep one side always facing the star (like the Moon keeps one side facing Earth). The flux computed in (a) is at the substellar point. Assuming that this planet does not possess an atmosphere, describe qualitatively how the surface temperature varies with location on the globe. Discuss which locations on the planet might be 'habitable' and 'uninhabitable'.

Solution:
Solar constant: 1.36×10^3 W m-2.

Problem 14-14

(a) The planet intercepts $\frac{10^{-3}}{1} \times \left(\frac{1}{0.03}\right)^2 \approx 1.11$ times as much energy per unit surface area as does the Earth.

Scale from Earth's equilibrium temperature of 255 K:

$$T_{\rm eq} = 255 \times \left[\frac{10^{-3}}{1} \times \left(\frac{1}{0.03}\right)^2\right]^{1/4} = 262\,\mathrm{K}.$$

So yes, this object does lie in the habitable zone if we assume that the planet has a similar albedo to its stars radiation as Earth and a modest greenhouse atmosphere like Earth's.

(b) The day side of this planet would continuous light from its star and be warmer than the planet average while the night side would never be heated by the star and be very cold. Regions near the terminator (boundary between day and night) and near the poles might have the most clement atmospheric conditions and be the "habitable zones" on such a planet.

Problem 7

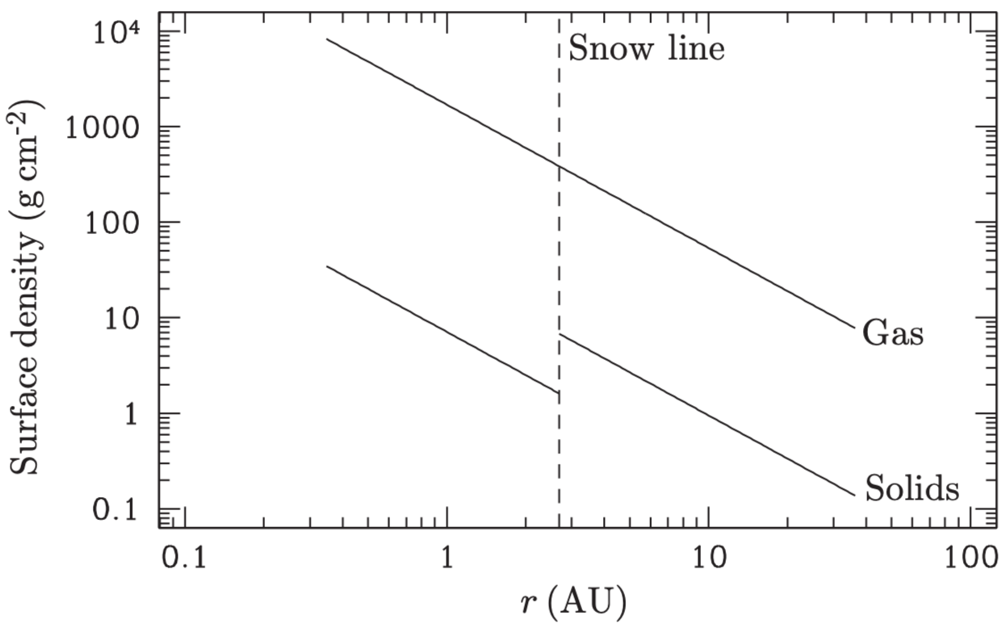
The mass of the disk of gas and dust that formed the Solar System is unknown. However, it is possible to use the observed masses, orbital radii and compositions of the planets to derive a lower limit for the amount of material that must have been present, together with a crude idea as to how that material was distributed with distance from the Sun. Such a structure around the protosun is called a Minimum Mass Solar Nebular, as we discussed in the class. The procedure is simple:

1. Starting from the observed (or inferred) masses of heavy elements (anything heavier than He) in the planets, augment the mass of each planet with enough hydrogen and helium to bring the augmented mixture to solar composition.
2. Divide the Solar System up into annuli, such that each annulus is centered on the current semi-major axis of a planet and extends halfway to the orbit of the neighboring planets.
3. Imagine spreading the augmented mass for each planet across the area of its annulus. This yields a characteristic gas surface density (in units g cm^{-2}) at the location of each planet.

Questions:

1. Follow the above steps to construct a MMSN. Plot the gas surface density as a function of radius.
2. What is the surface density of the disk at 1 AU?
3. Calculate the total mass of this disk out to 30 AU.
   Hint 1: The composition of planets, particularly the mass in "heavy elements" in the outer giant planets, can be found in the textbook (or Wikipedia)
   Hint 2: Very crudely, the terrestrial planets are made of refractories (metals, silicates, etc). In the pre-solar disk, 0.3% of the mass is in such material.
   Hint 3: The heavy elements in the outer 4 giant planets contain both refractories and volatiles (H2O, CH4, etc). In the pre-solar disk, 1.2% of the mass is in such material.

Solution: (1) The terrestrial planets have almost no H and He. For giant planets, their compositions can be found in C.8.1.3, C.8.2.2, and C.8.3.2.



To answer the question, students only need to show the gas line. Other lines are there to guide interpretation.
(2): read from the above table, it's about 3000 g cm^{-2}
(3): on the order of 0.01 solar mass.

Problem 8

Let's assume that at stellocentric R = 1 AU in the Minimum Mass Solar Nebular, the temperature is 150 K at all heights. Estimate the density at (R = 1 AU, z = 0.05 AU) in the disk. You will need to use your results from the above problem.

Solution: Using the parameters provided, we can calculate the scale height H at 1 AU to be 0.025 AU. Σ(1 AU) = 3000 g cm^{-2}. ρ(1 AU, z=0) = Σ_{MMSN} / sqrt(2PI) / H = 3.2x10-9 g cm^{-3}. . ρ(1 AU, z=0.05 AU) = ρ(1 AU, z=0 AU) × exp($-z^2/(2\times H^2)$) = 4.2×10^{-10} g cm^{-3}.

Short questions

- Is the solar system the only planetary system in the universe?
  - No
- When did people discover the first planet orbiting a normal star other than the Sun?
  - 1995
- How many planets have people discovered so far
  - A few thousands
- How common are planets in our galaxy
  - Very common; nearly every star has planets
- Is the Solar system common or not?
  - Unclear at the moment. There are many planetary systems quite different from the solar system.