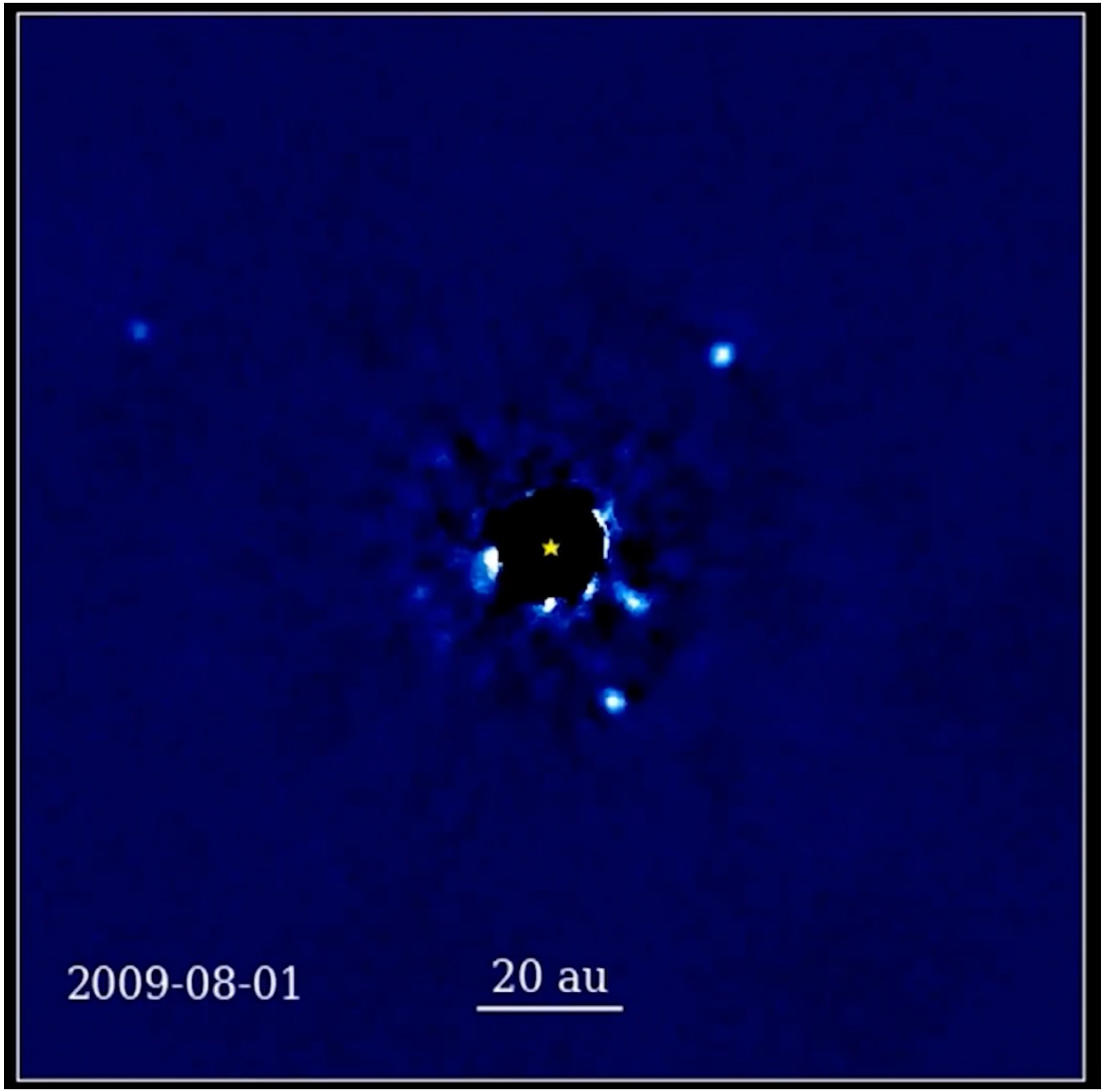
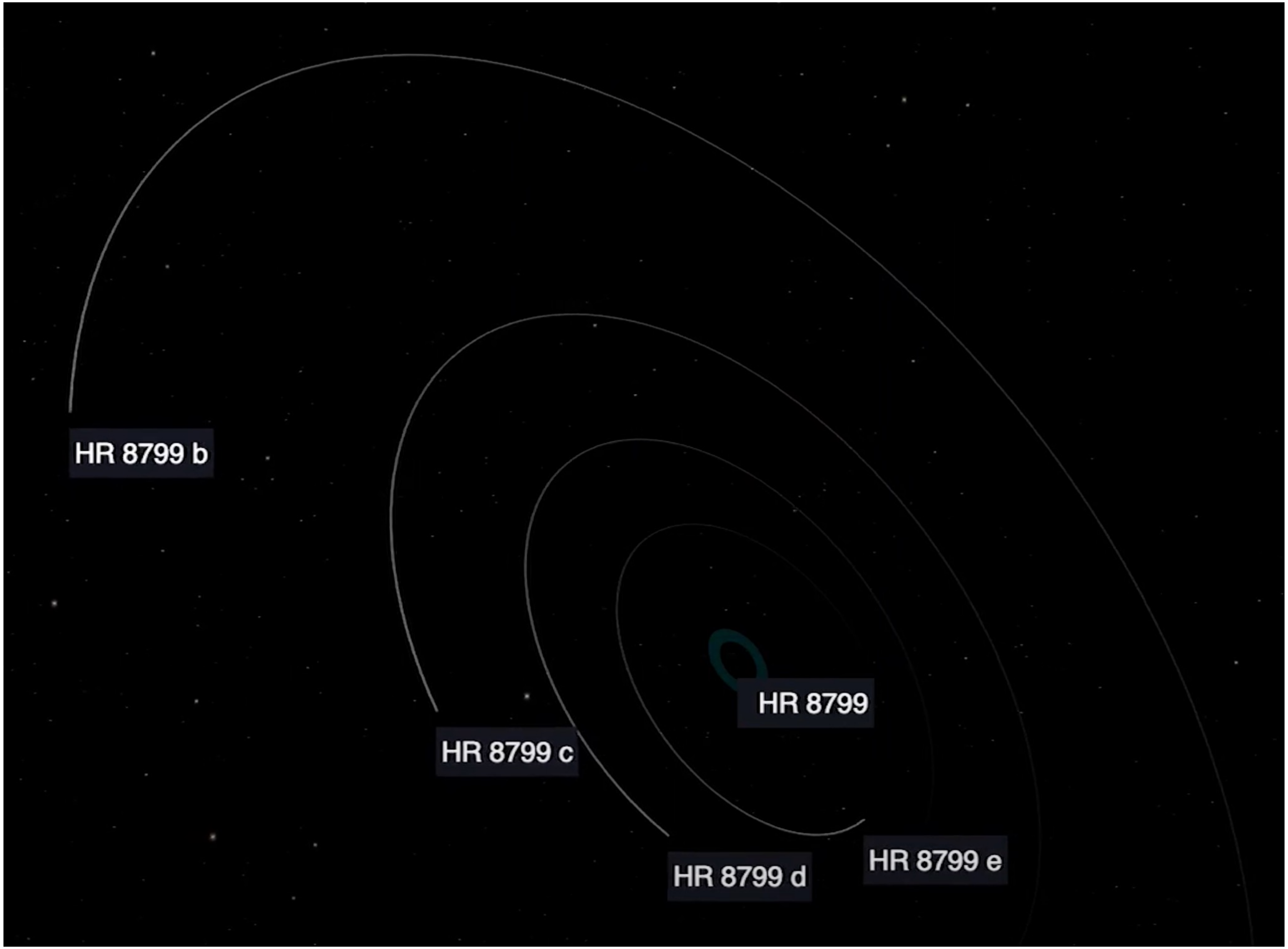
Chapter 14: Exoplanet



A decade of orbital motion
Jason Wang
Christian Marois


HR 8799 b
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HR 8799 e
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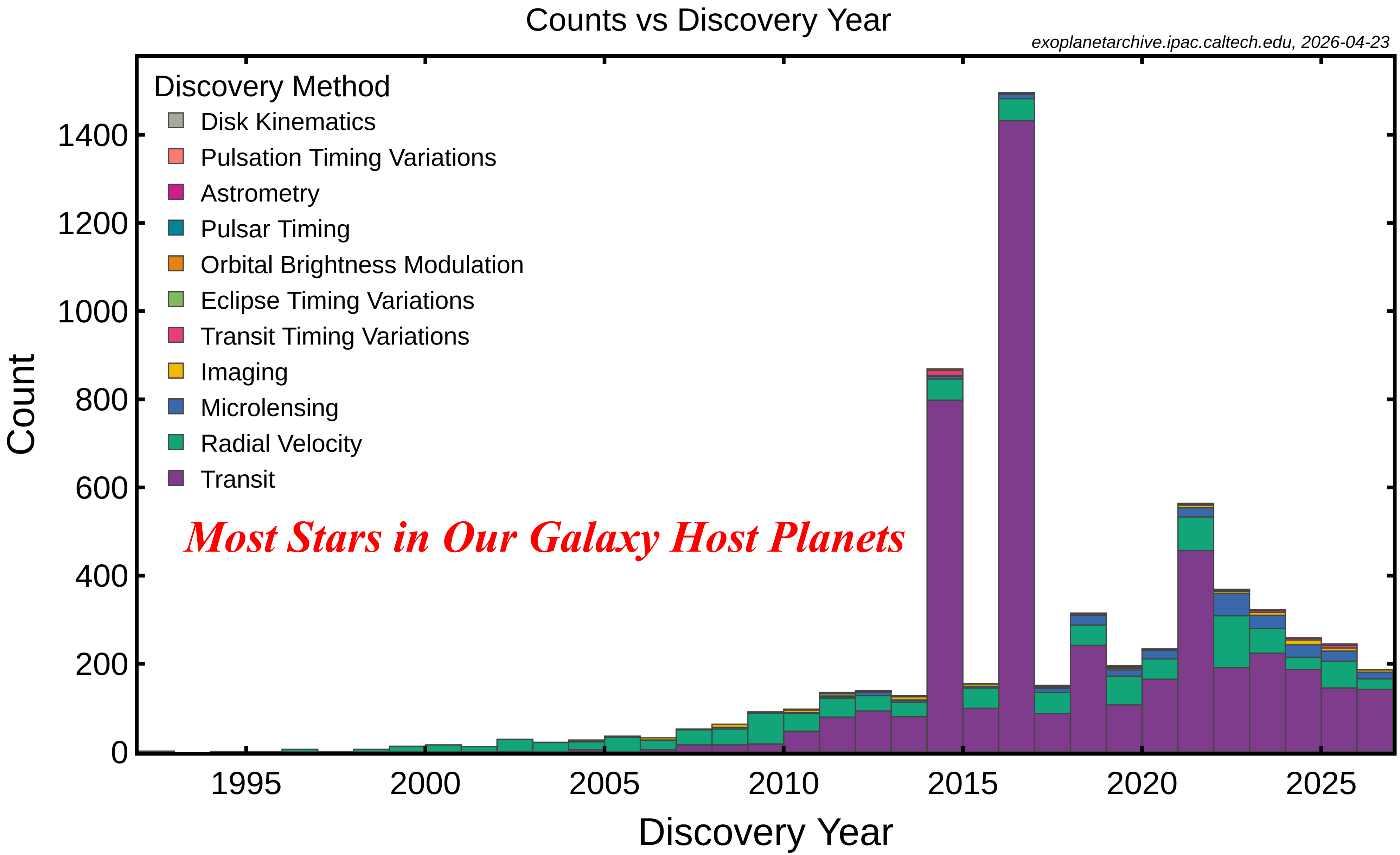
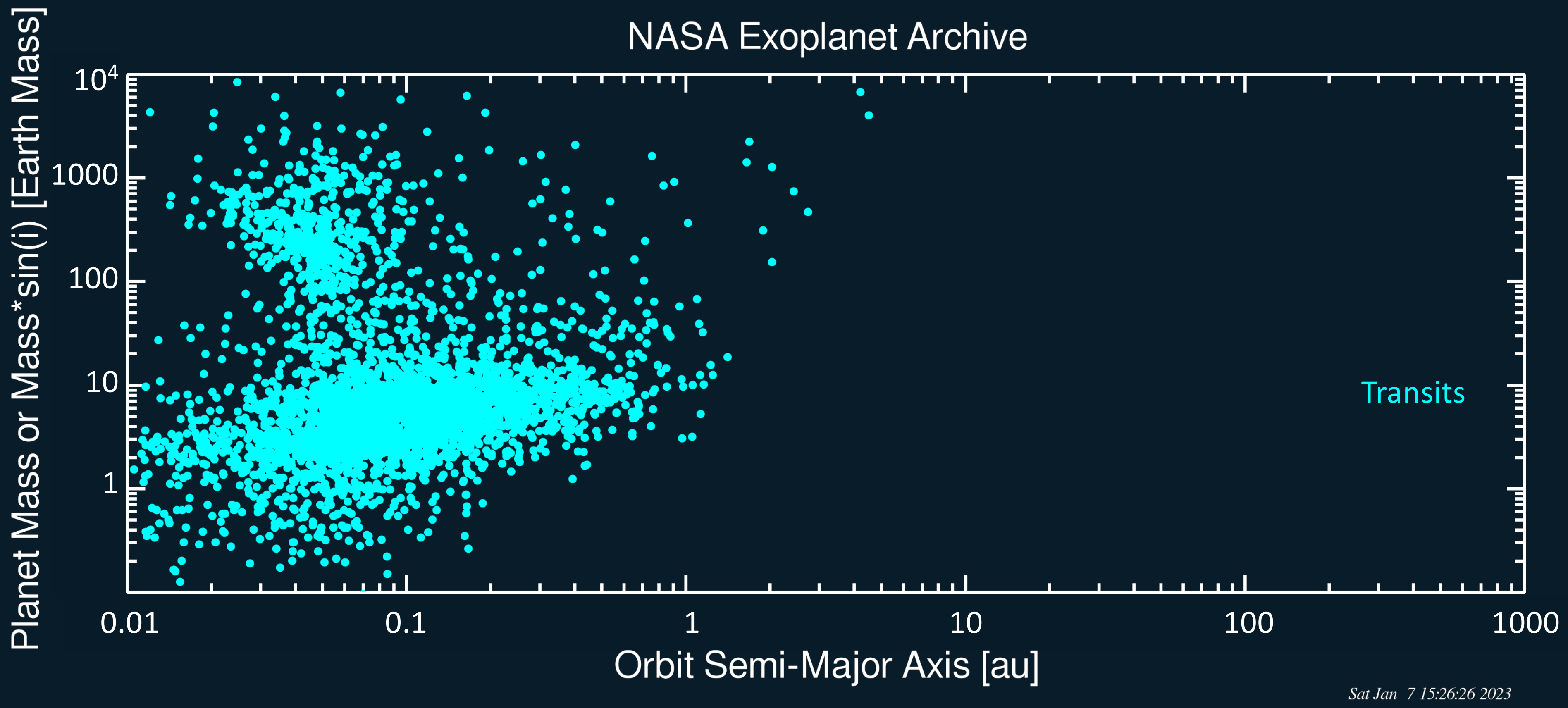


exoplanetarchive.ipac.caltech.edu, 2026-04-23

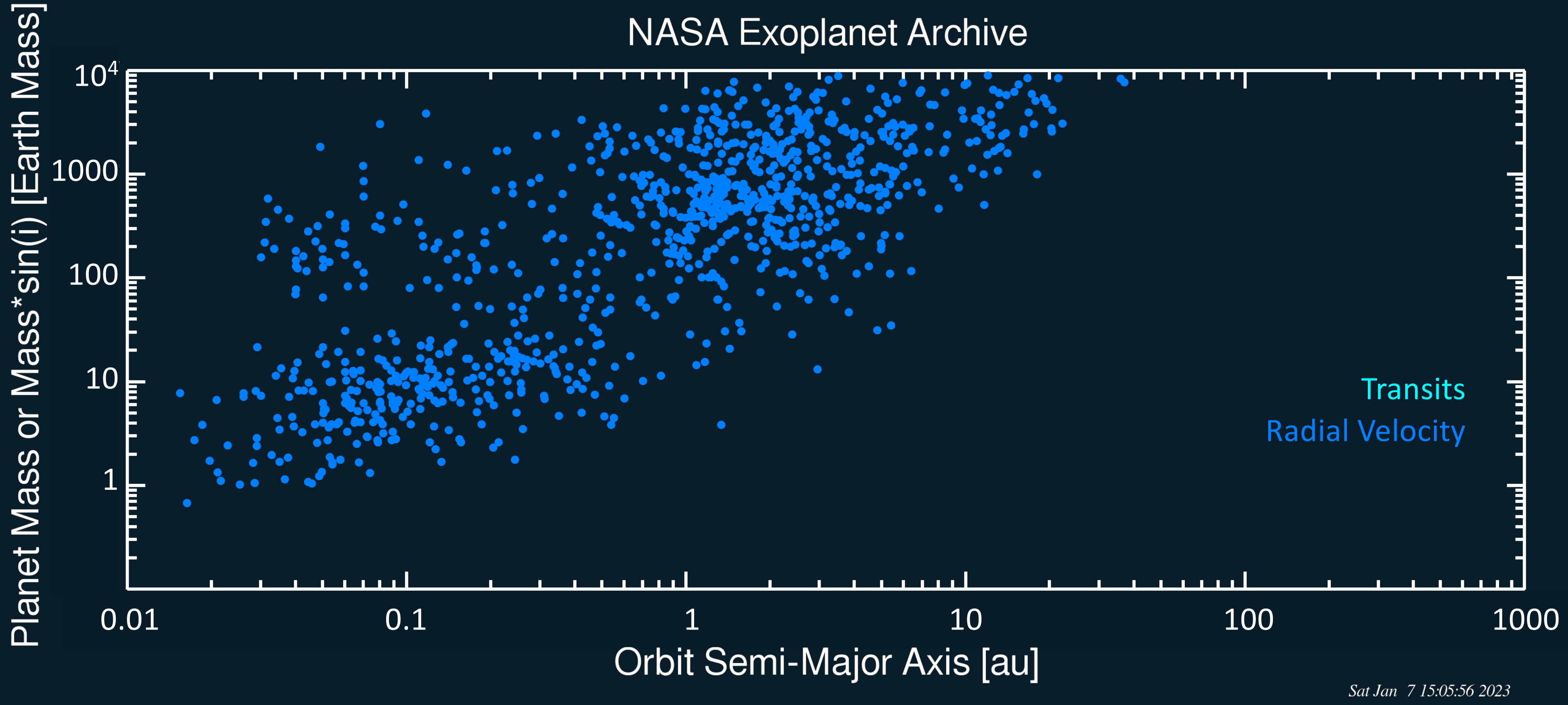
Most Stars in Our Galaxy Host Planets

5000 exoplanets… and counting!



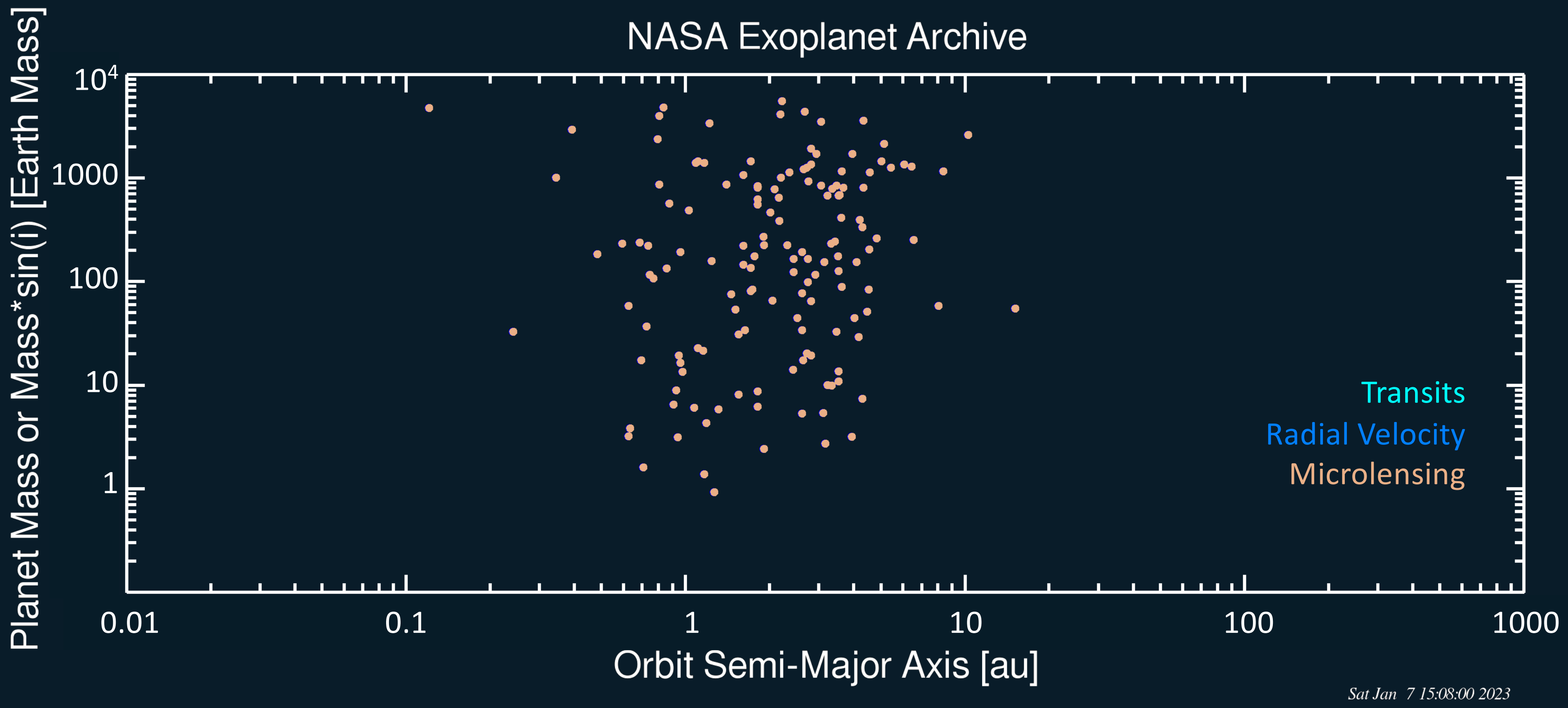
Jessie Christiansen

5000 exoplanets… and counting!



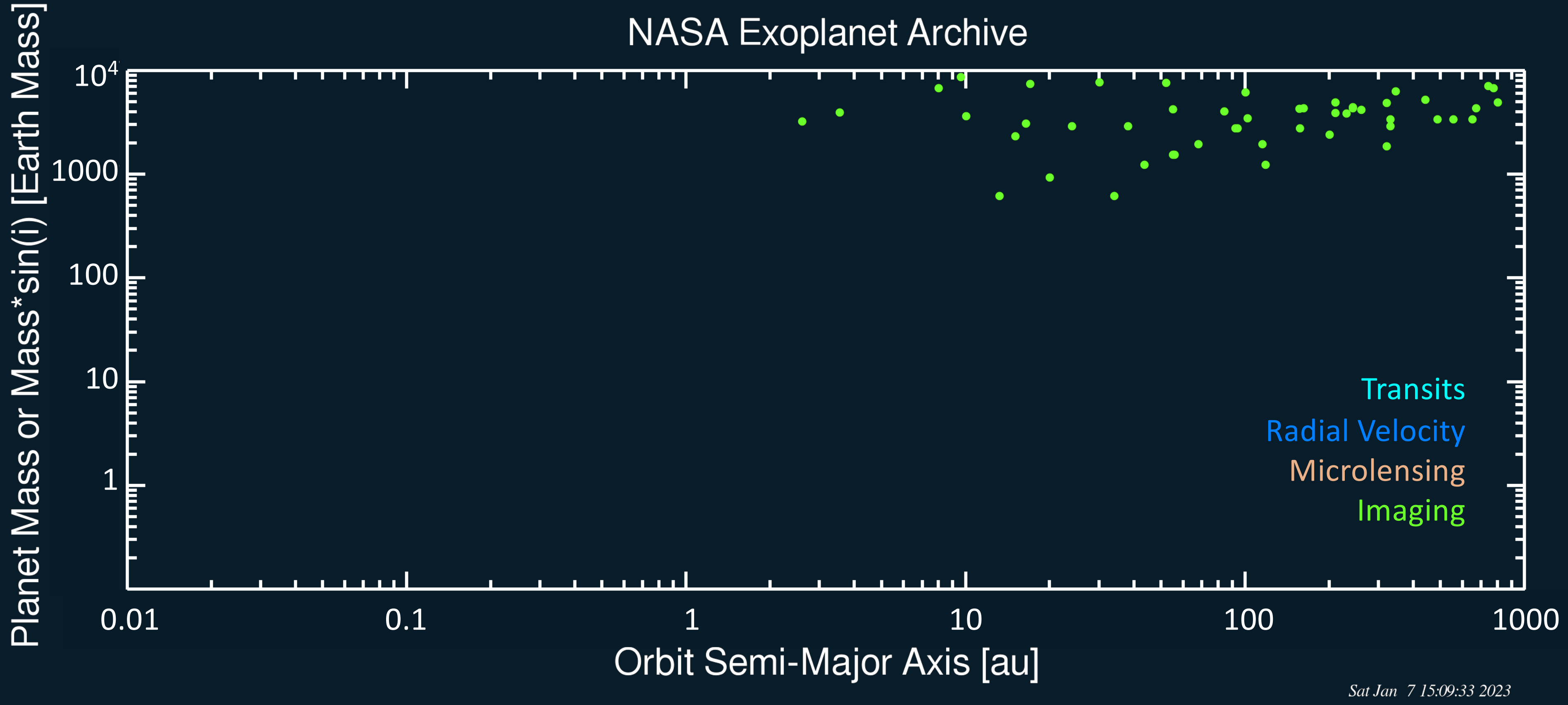
Jessie Christiansen

5000 exoplanets… and counting!



Jessie Christiansen

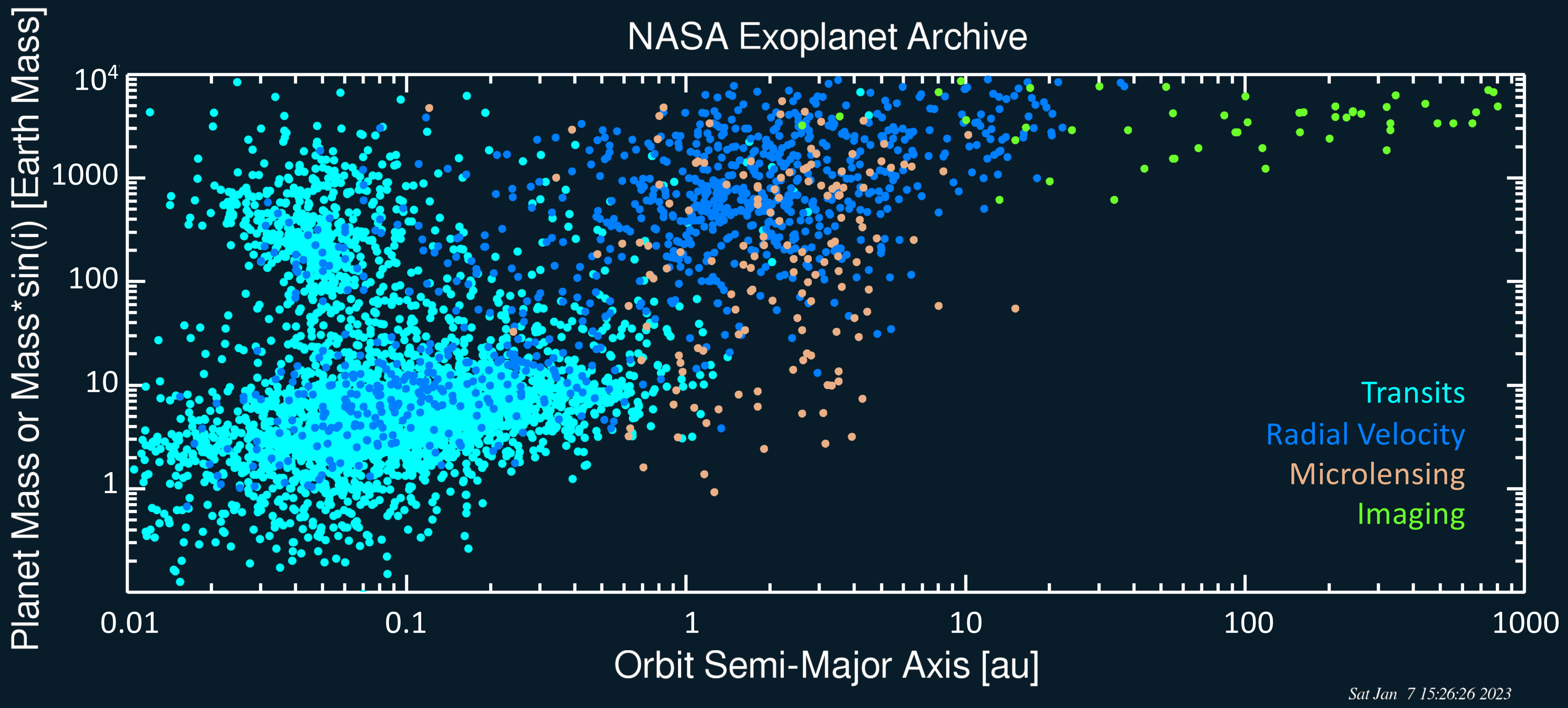
5000 exoplanets… and counting!



Sat Jan 7 15:09:33 2023

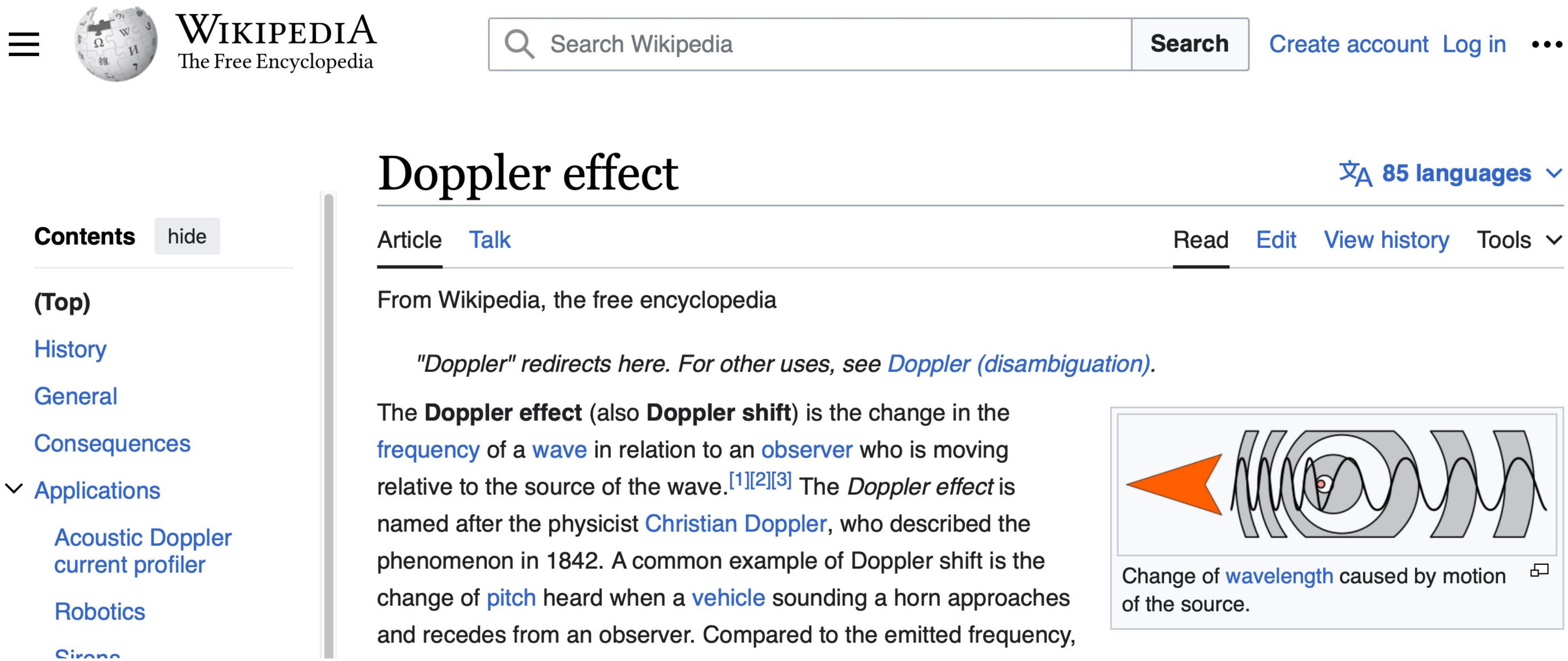
Jessie Christiansen

5000 exoplanets… and counting!



Jessie Christiansen

Radial Velocity



Doppler effect

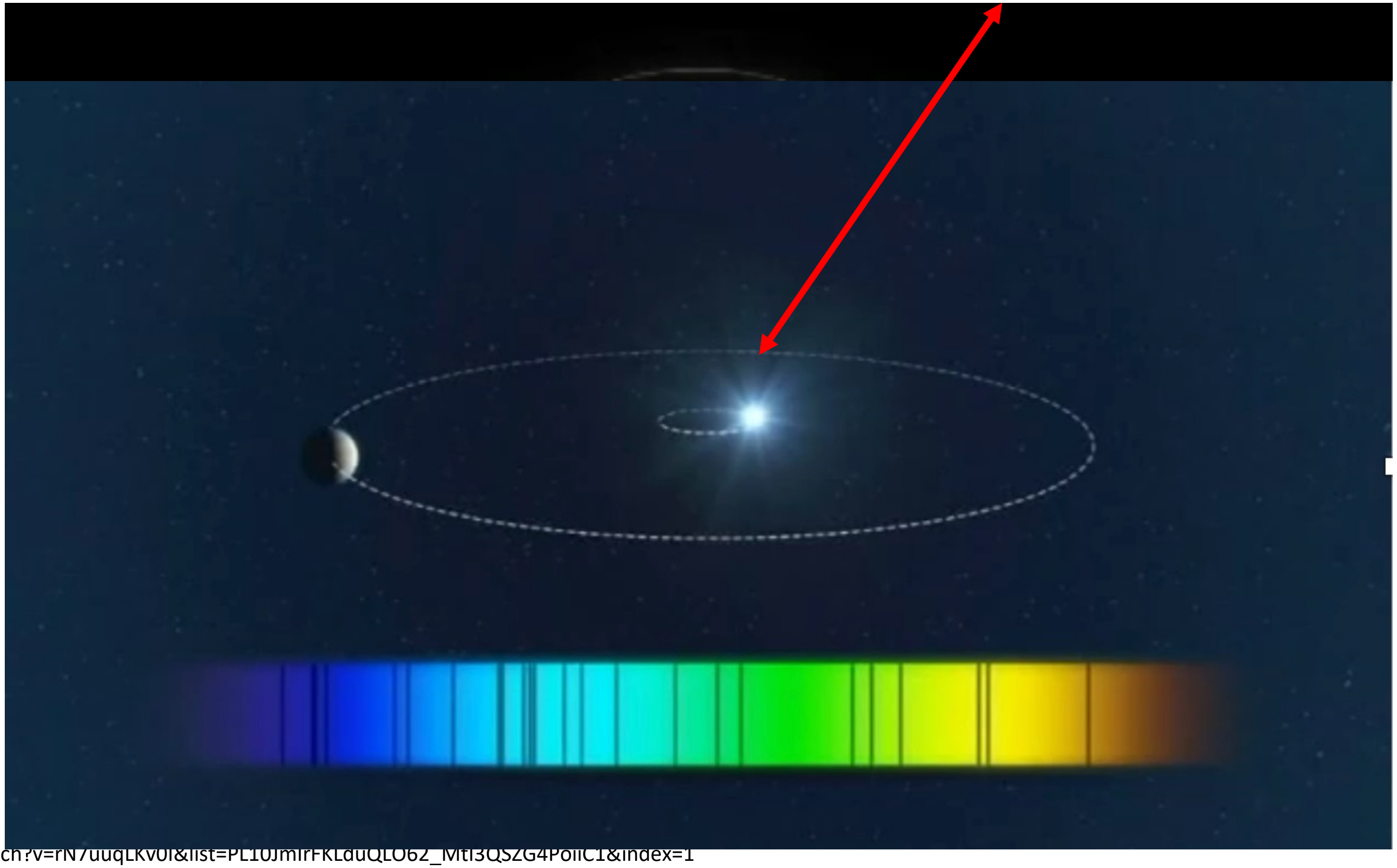
From Wikipedia, the free encyclopedia

"Doppler" redirects here. For other uses, see Doppler (disambiguation).

The **Doppler effect** (also **Doppler shift**) is the change in the frequency of a wave in relation to an observer who is moving relative to the source of the wave.[1][2][3] The *Doppler effect* is named after the physicist Christian Doppler, who described the phenomenon in 1842. A common example of Doppler shift is the change of pitch heard when a vehicle sounding a horn approaches and recedes from an observer. Compared to the emitted frequency,

Change of wavelength caused by motion of the source.

Radial Velocity

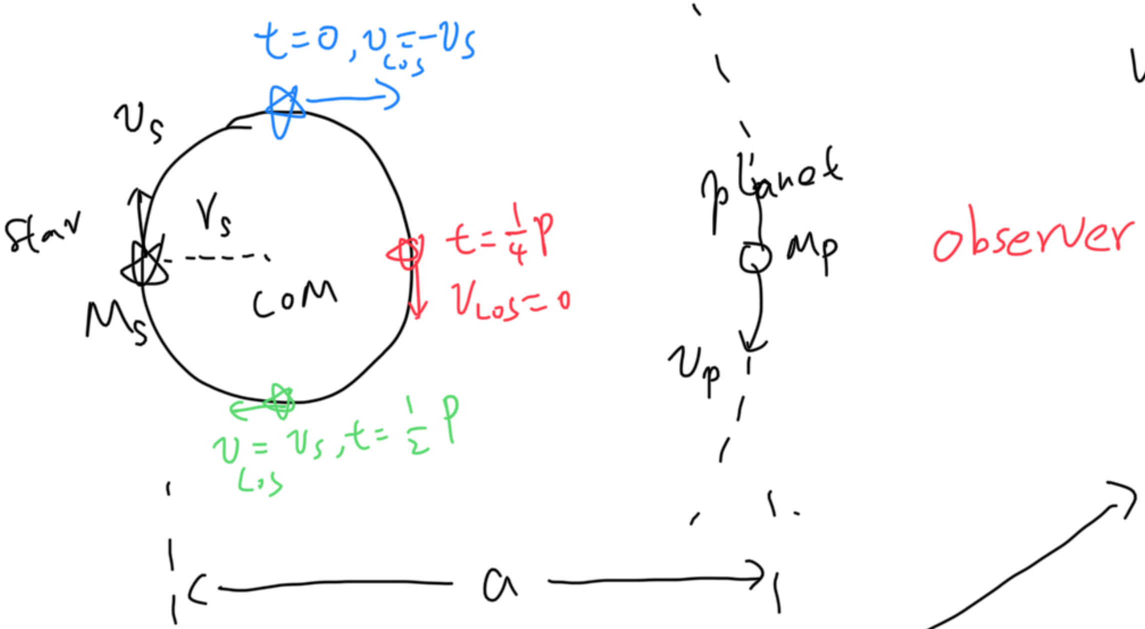


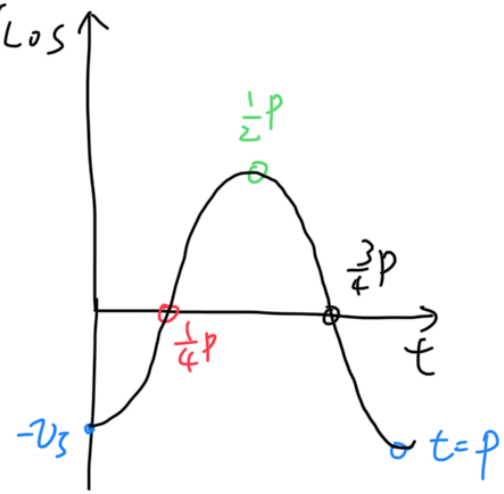
radial velocity of the star

https://www.youtube.com/watch?v=rN7uuqLKv0I&list=PL1UJmIrFKLduQLO62_MtI3QSZG4PoiiC1&index=1

← radial velocity +

Circular orbit, $e = 0$





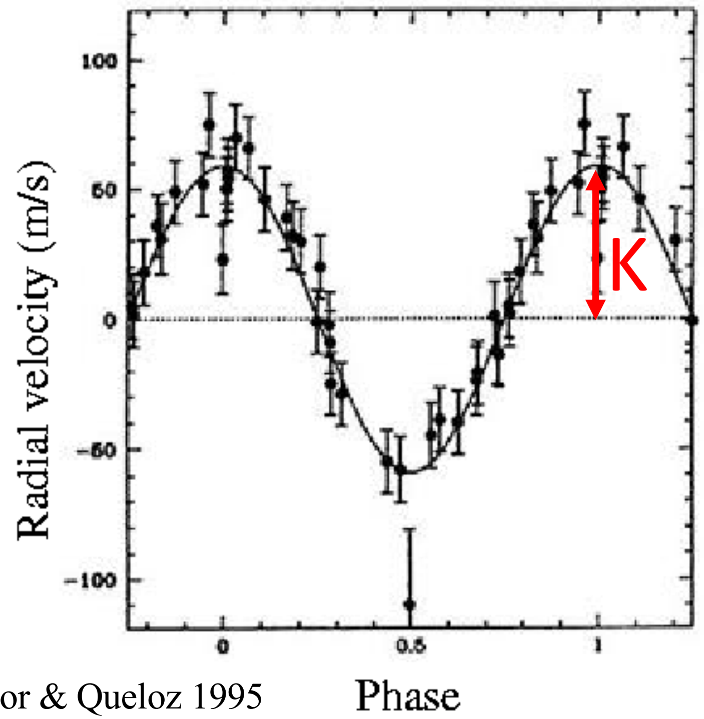
$$F = ma$$

$$\frac{G M_s M_p}{a^2} = M_s \frac{v_s^2}{r_s} , \qquad M_s r_s = M_p (a - r_s) \Rightarrow r_s = \frac{M_p a}{M_s + M_p}$$

$$\frac{G M_p}{a^2} = \frac{v_s^2 (M_s + M_p)}{M_p a} \Rightarrow \boxed{v_s = \sqrt{\frac{G M_p^2}{(M_s + M_p) a}}}$$

Radial Velocity

First exoplanet around a sun-like star
Nobel prize 2019



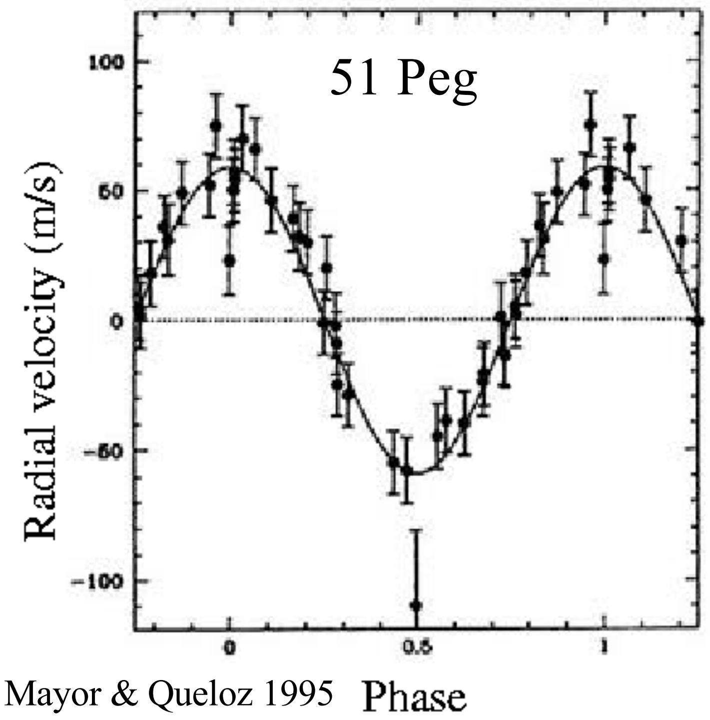
Mayor & Queloz 1995

Semi-amplitude K

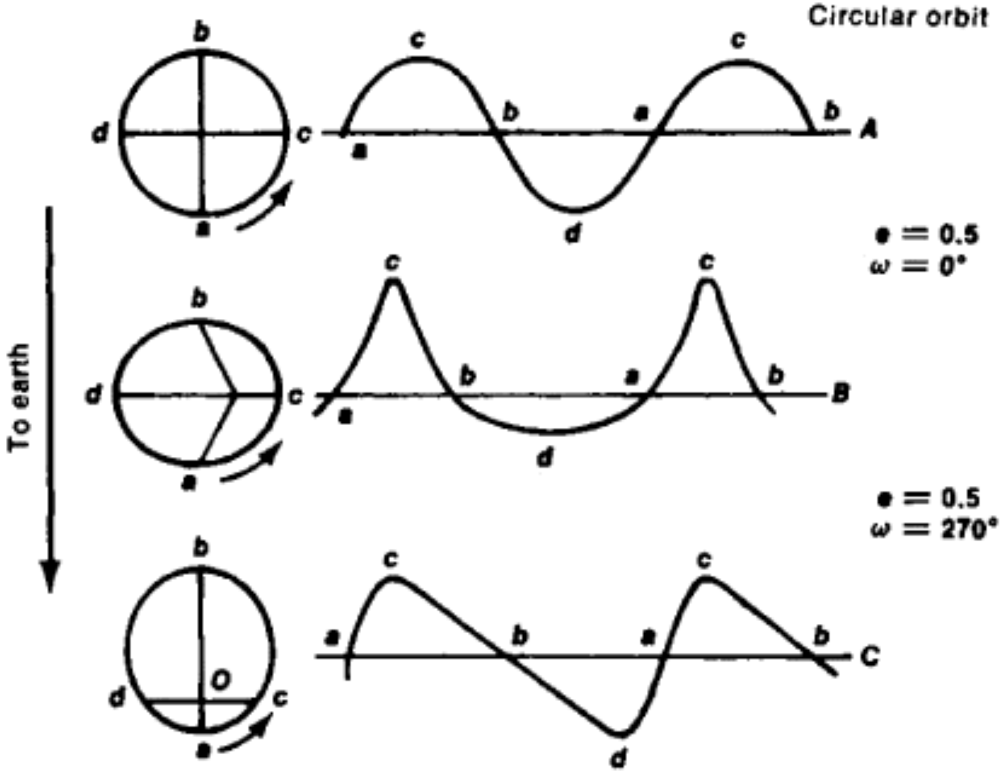
$$K = \left(\frac{2\pi G}{P_{\rm orb}}\right)^{1/3} \frac{M_{\rm p} \sin i}{(M_\star + M_{\rm p})^{2/3}} \frac{1}{\sqrt{1-e^2}}$$

$$= \left(\frac{G}{a}\right)^{1/2} \frac{M_{\rm p} \boxed{\sin i}}{(M_\star + M_{\rm p})^{1/2}} \frac{1}{\boxed{\sqrt{1-e^2}}}, \quad (14.1)$$

RV constraints on eccentricity

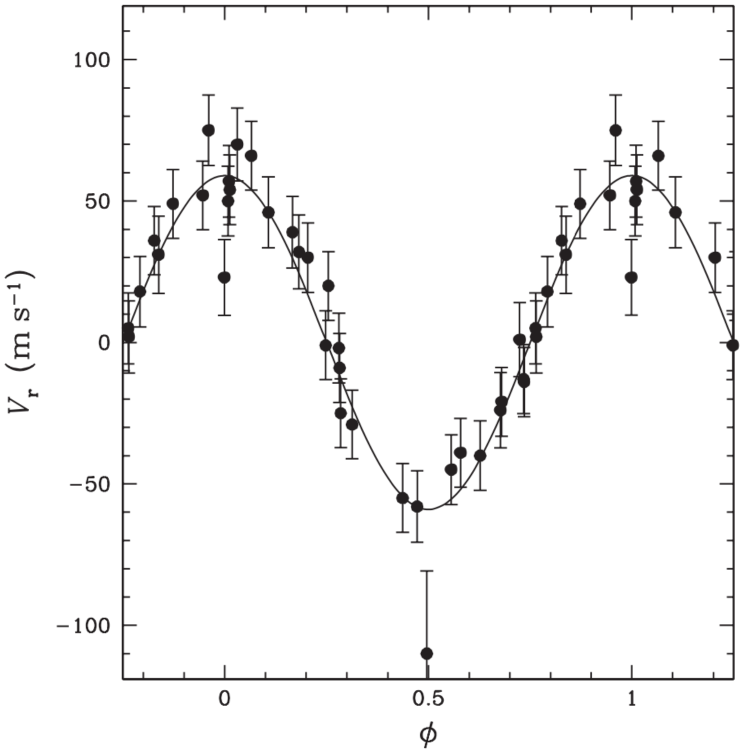


Mayor & Queloz 1995



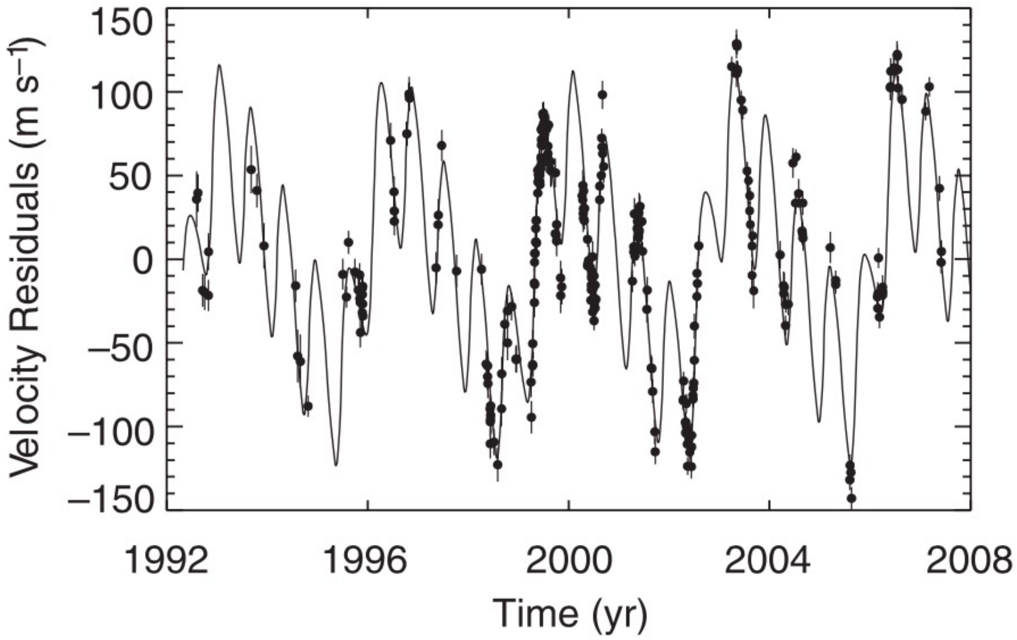
RV multi-planet systems

51 Peg



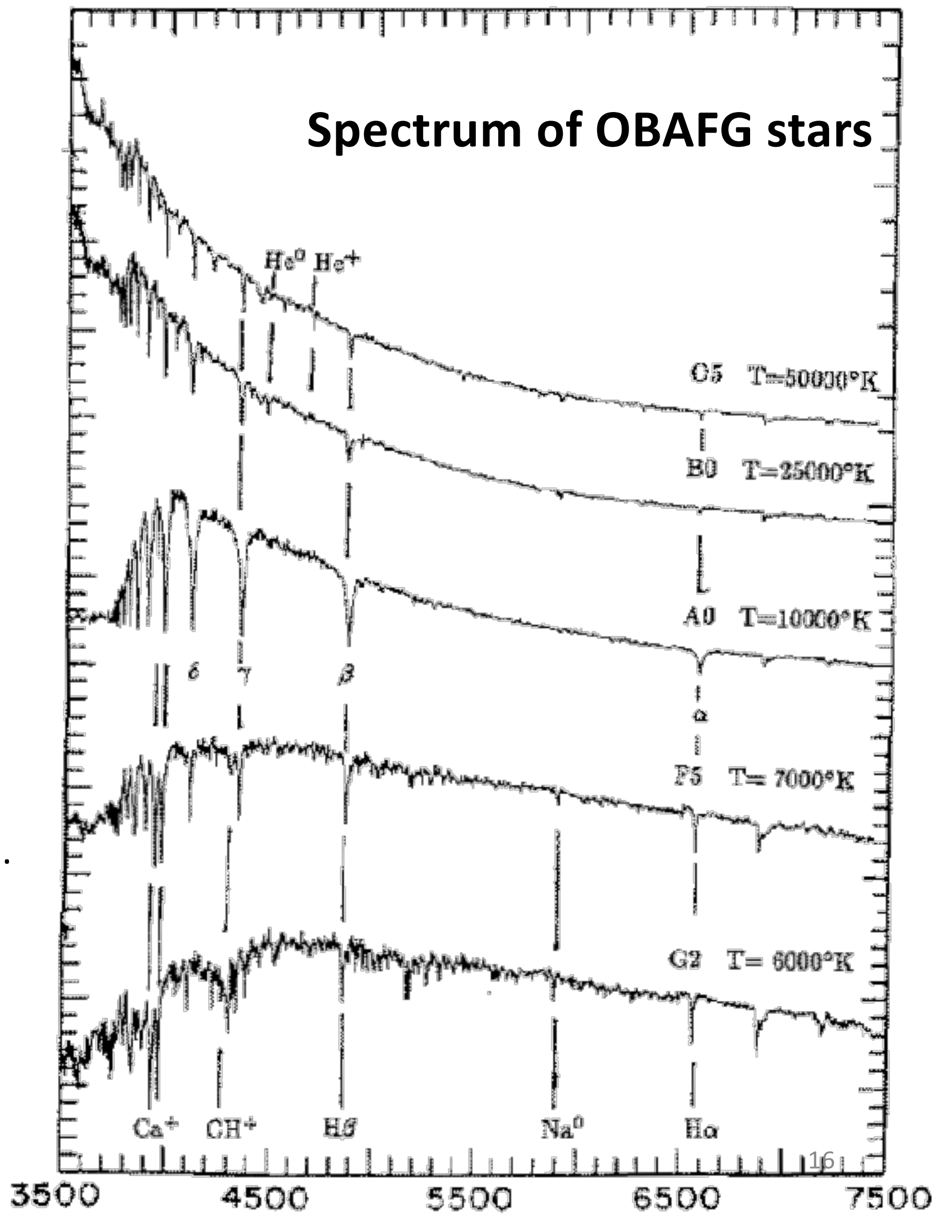
One planet

υ Andromadae



Two planets

Radial Velocity


Spectrum of OBAFG stars

RV measurements require a large number of narrow spectral lines. Thousands of such lines of depth > 1% are present in most main sequence stars similar to the Sun and cooler (FGK).

The hottest stars (ABO) have few features in their optical spectra.

Supplemental reading: stellar classification https://en.wikipedia.org/wiki/Stellar_classification

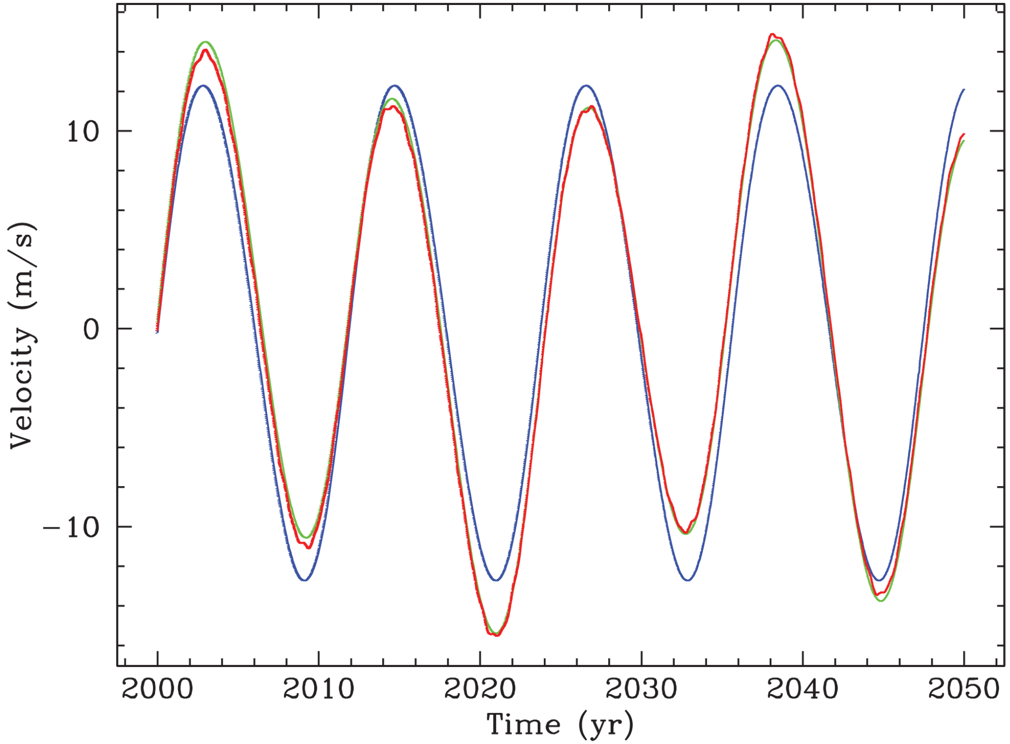
Radial Velocity

Figure 14.1 COLOR PLATE Velocity variations of the Sun in response to Jupiter (nearly sinusoidal *narrow blue curve*), Jupiter plus Saturn (*faint green curve*) and all eight planets plus Pluto (*thick red curve*). Jupiter's tug dominates the variations, with Saturn having much less influence than Jupiter but still far more than all of the remaining planets combined. The pull of Earth and Venus is evident in the short-period variations seen in the *thick red curve*. (Courtesy Elisa V. Quintana)

Jupiter

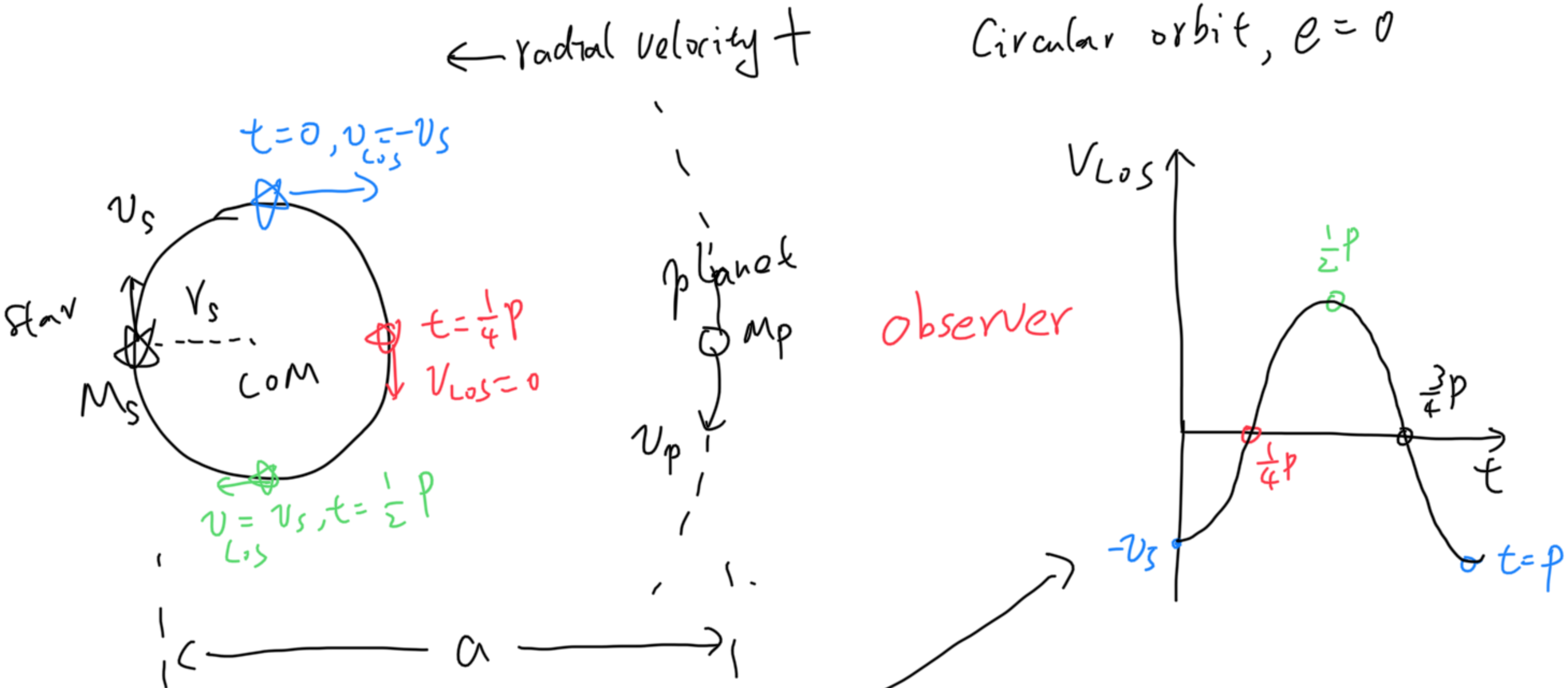
Jupiter + Saturn

All planets



Astrometry

https://www.youtube.com/watch?v=l46-8PvT44Y



$$F = ma$$

$$\frac{G M_s M_p}{a^2} = M_s \frac{v_s^2}{r_s}, \qquad M_s r_s = M_p (a - r_s) \Rightarrow r_s = \frac{M_p a}{M_s + M_p}$$

$$\frac{G M_p}{a^2} = \frac{v_s^2 (M_s + M_p)}{M_p a} \Rightarrow \boxed{v_s = \sqrt{\frac{G M_p^2}{(M_s + M_p) a}}}$$

$$\boxed{\Delta\theta \le \frac{M_\mathrm{p}}{M_\star}\frac{a}{r_\odot},}$$

Astrometry

Jupiter

Jupiter + Saturn

All planets

Solar disk

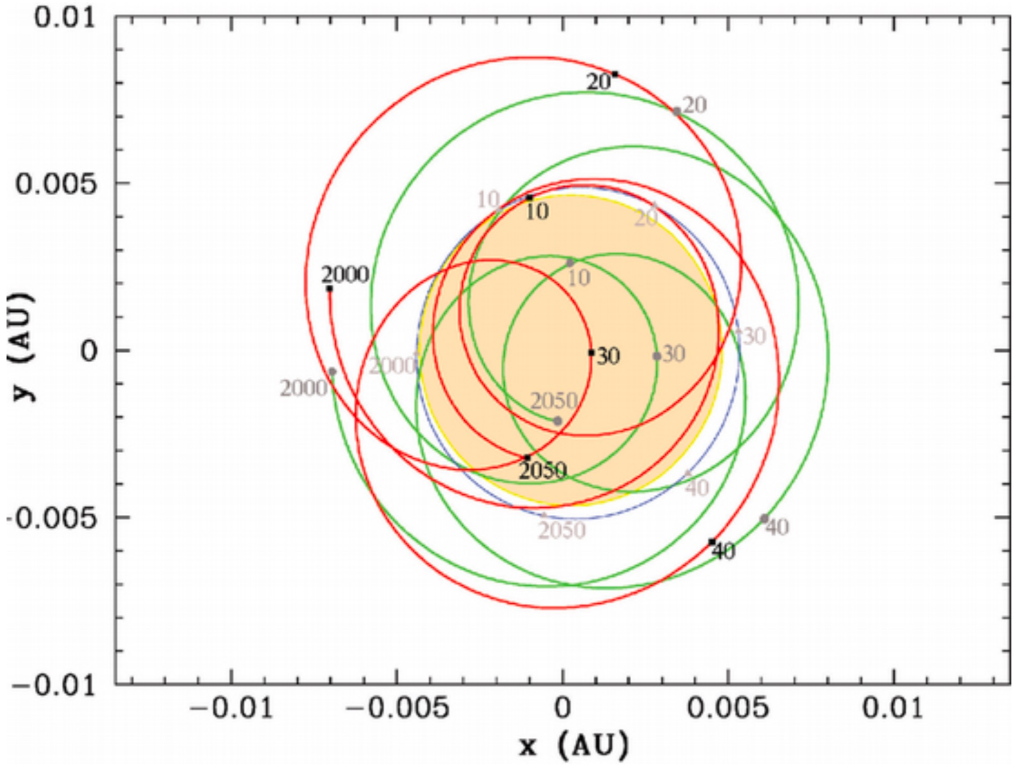


Figure 14.2 COLOR PLATE Motion of the Sun during the first half of the twenty-first century in response to Jupiter (*narrow blue ellipse, faint dates*), Jupiter plus Saturn (*light green curve*) and all eight planets plus Pluto (*thick dark red curve, dark dates*). The solar disk (*shaded yellow*) is shown for comparison. The Sun moves counterclockwise in this perspective, completing slightly less than one trip around the elliptical curve per decade. Jupiter's tug dominates the variations on short timescales, but since $\Delta\theta$ increases with a, Saturn, Uranus and Neptune have more influence on the Sun's position than they do on the Sun's velocity (see Fig. 14.1). The amplitude of the Sun's motion induced by the terrestrial planets is very small. (Courtesy E. V. Quintana)

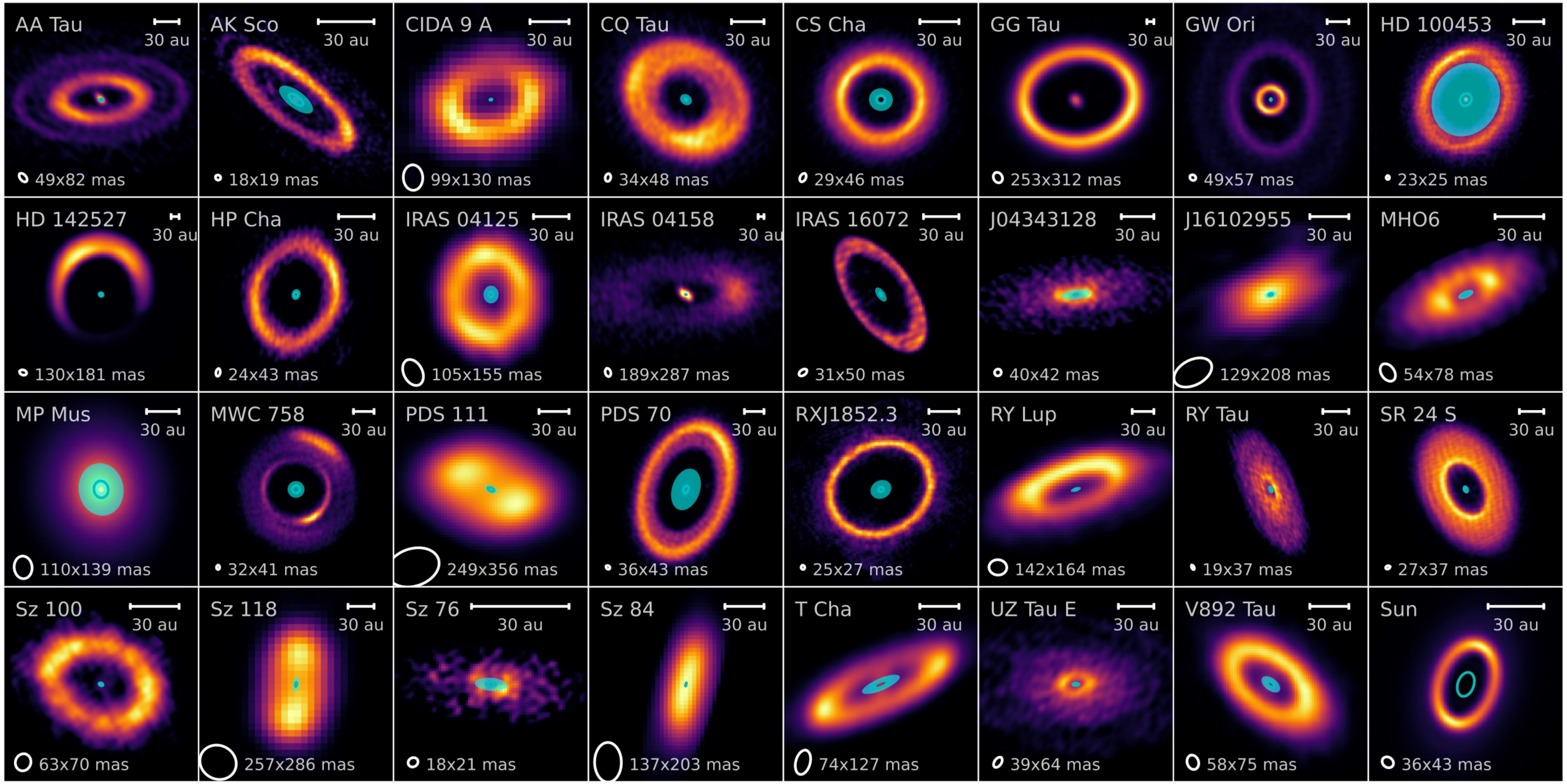
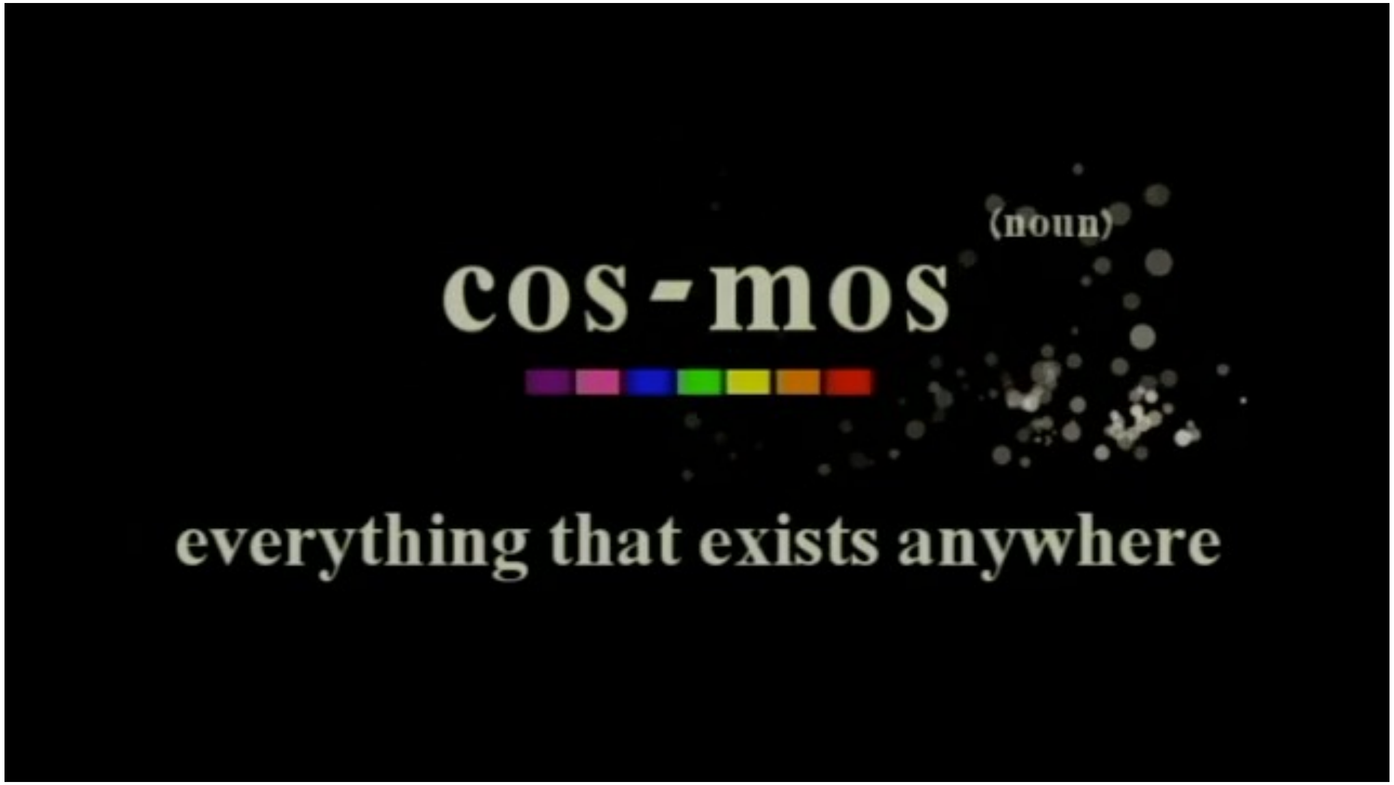


Fig. 1. ALMA continuum images of the sample of 31 discs with inner dust cavities (‘transition discs’) for which we find a significant proper motion anomaly ($|\Delta\mu|/\sigma_{|\Delta\mu|} \geq 3$) indicative of the presence of companions. Assuming one companion dominates the proper motion anomaly (Sect. 2), the solid cyan line indicates the 50th percentile of the companion location (the cyan coloured areas are the 10th and 90th percentiles) as derived in Sect. 3.1 (see Figs. 2 and A.2, exception is GG Tau, whose companion could not be modelled). Bottom-right corner: Image of the Sun, for reference, as it is predicted to look at 1 Myr (Bergez-Casalou et al. 2022) with Jupiter’s orbit in cyan. The ALMA synthesised beams are included at the bottom left of each panel. Non-detections can be seen in Fig. A.1.

Transit



https://www.youtube.com/watch?v=FVGnspcYQ4Q&list=PL10JmIrFKLduQLO62_MtI3QSZG4PoiiC1&index=3

Transit: Photometry

Transit depth

$$\frac{\Delta\mathcal{F}}{\mathcal{F}} = \left(\frac{R_{\rm p}}{R_\star}\right)^2 .$$

Transit probability

$$\mathcal{P}_{\rm tr} = \frac{R_\star + R_{\rm p}}{a(1 - e^2)}.$$

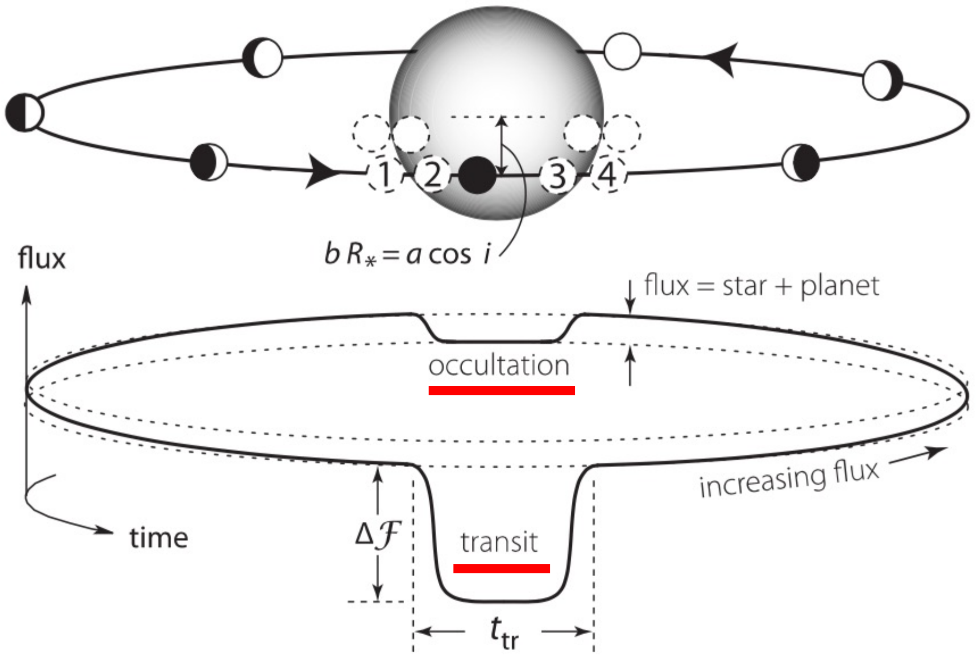


Figure 14.3 Schematic illustration of a transiting planet on a circular orbit. The transit begins at **first contact**, where the planet is at position (1) in the *upper diagram*. The entire disk of the planet blocks light from the star from the time of **second contact** (2) through that of **third contact** (3), and the transit concludes at **fourth contact** (4). (Adapted from Perryman 2018)

Solar system planet transits

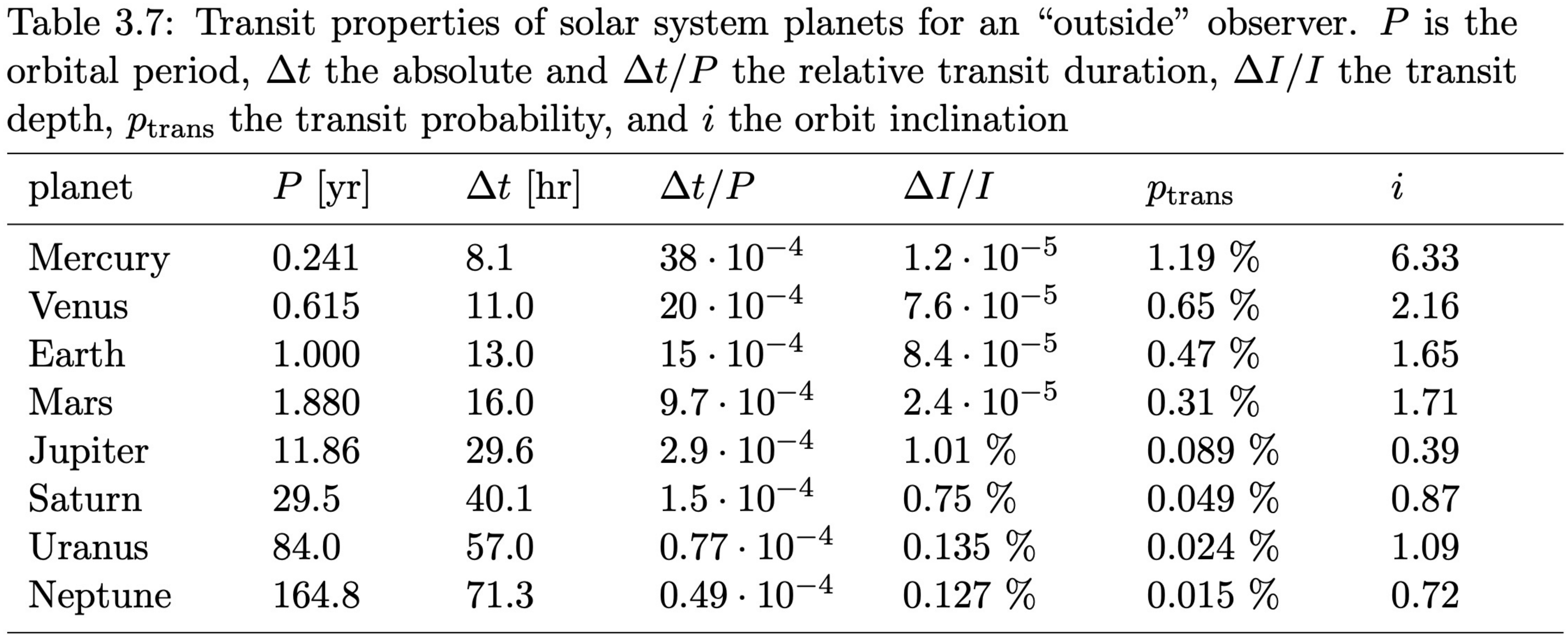


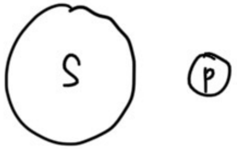
Table 3.7: Transit properties of solar system planets for an "outside" observer. P is the orbital period, Δt the absolute and $\Delta t/P$ the relative transit duration, $\Delta I/I$ the transit depth, p_{trans} the transit probability, and i the orbit inclination

planet	P [yr]	Δt [hr]	$\Delta t/P$	$\Delta I/I$	p_{trans}	i
Mercury	0.241	8.1	$38 \cdot 10^{-4}$	$1.2 \cdot 10^{-5}$	1.19 %	6.33
Venus	0.615	11.0	$20 \cdot 10^{-4}$	$7.6 \cdot 10^{-5}$	0.65 %	2.16
Earth	1.000	13.0	$15 \cdot 10^{-4}$	$8.4 \cdot 10^{-5}$	0.47 %	1.65
Mars	1.880	16.0	$9.7 \cdot 10^{-4}$	$2.4 \cdot 10^{-5}$	0.31 %	1.71
Jupiter	11.86	29.6	$2.9 \cdot 10^{-4}$	1.01 %	0.089 %	0.39
Saturn	29.5	40.1	$1.5 \cdot 10^{-4}$	0.75 %	0.049 %	0.87
Uranus	84.0	57.0	$0.77 \cdot 10^{-4}$	0.135 %	0.024 %	1.09
Neptune	164.8	71.3	$0.49 \cdot 10^{-4}$	0.127 %	0.015 %	0.72

S. Quanz Lecture notes, ETH Zurich

Ⓐ Ignore the day/night sight difference on a planet

①



star + planet

②

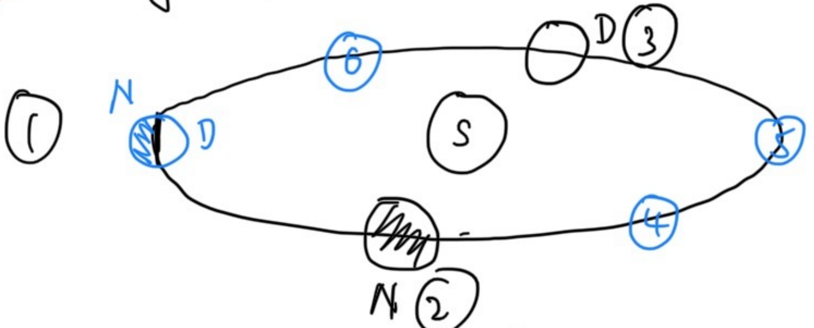
star×(1−depth)+planet

③

star

$$\text{①} - \text{③} = \text{planet} = 4\pi R_p^2(\sigma T_p^4) \Rightarrow \boxed{T_p}$$

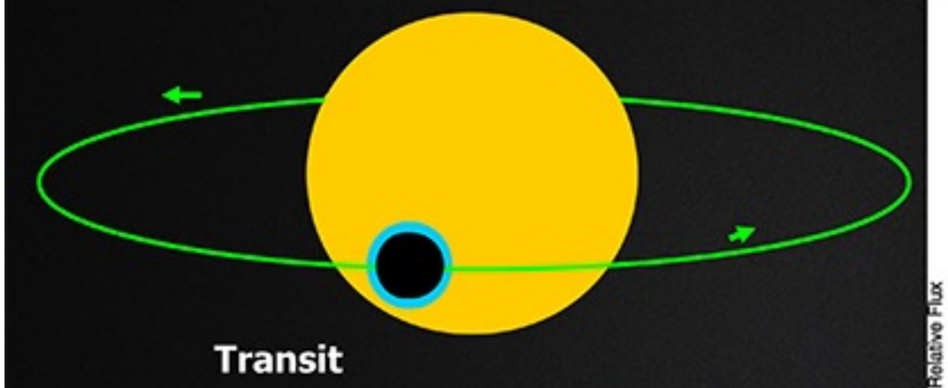
Ⓑ Day/night side into account



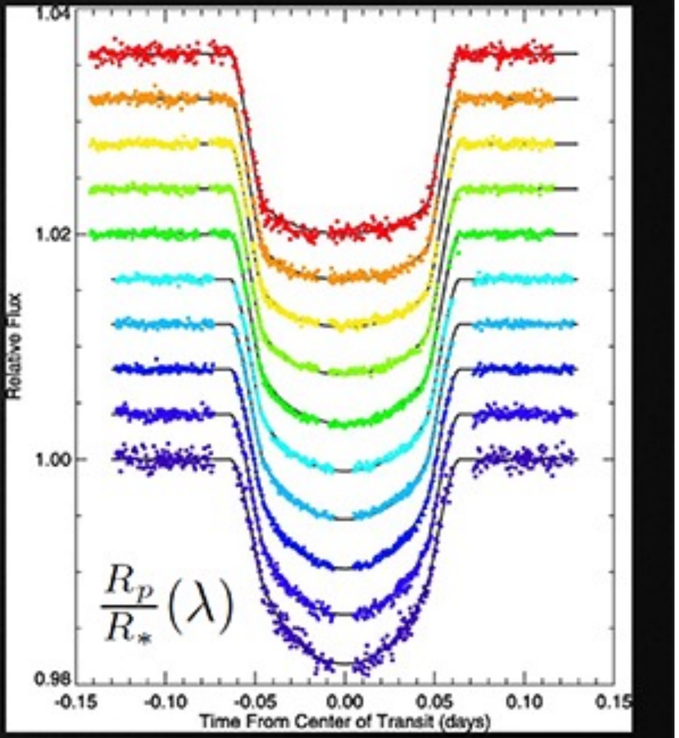
$$\text{③} - \text{②} = \underbrace{[\text{star} + \text{planet (day)}]}_{\text{③}} - \underbrace{[\text{star} + \text{planet (night)}]}_{\text{②}} = \underline{\text{plant (day)} - \text{planet (night)}}$$

①, ②, ③ ... ⇒ longitudinal map

Transmission spectroscopy

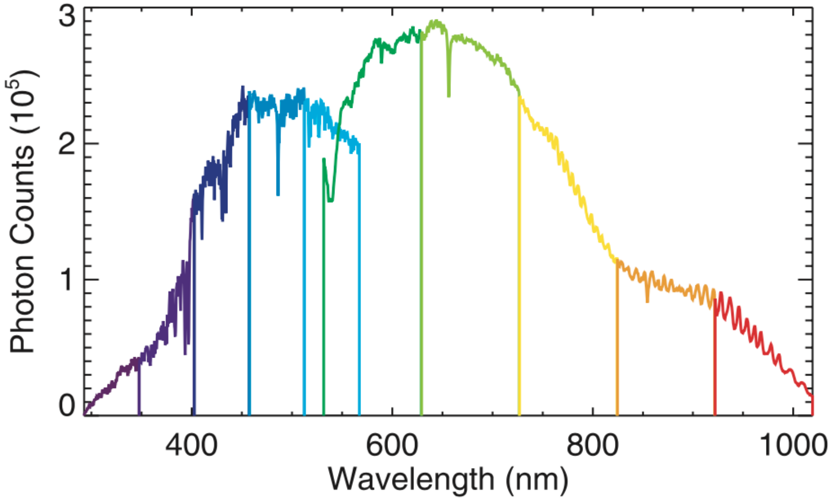


$$\frac{\Delta f}{f} = \left(\frac{R_p}{R_*}\right)^2$$



HD 209458b from 290–1030 nm (Knutson et al. 2007)

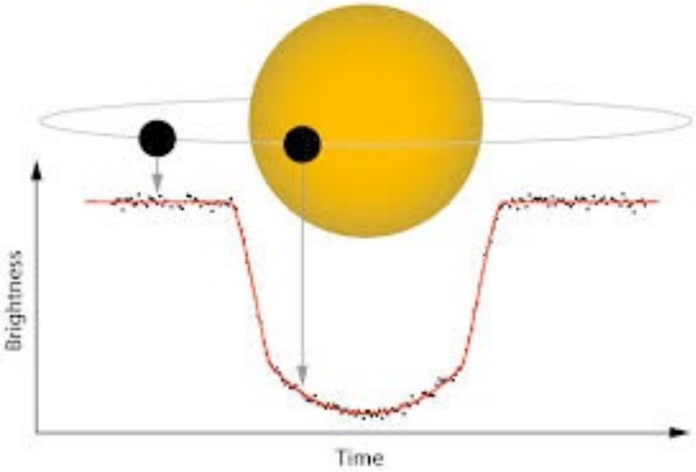
Different colors are at different wavelengths



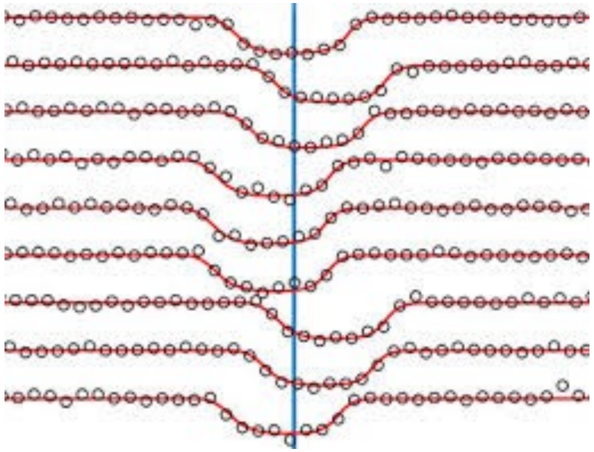
Transit: Timing Variation

NASA

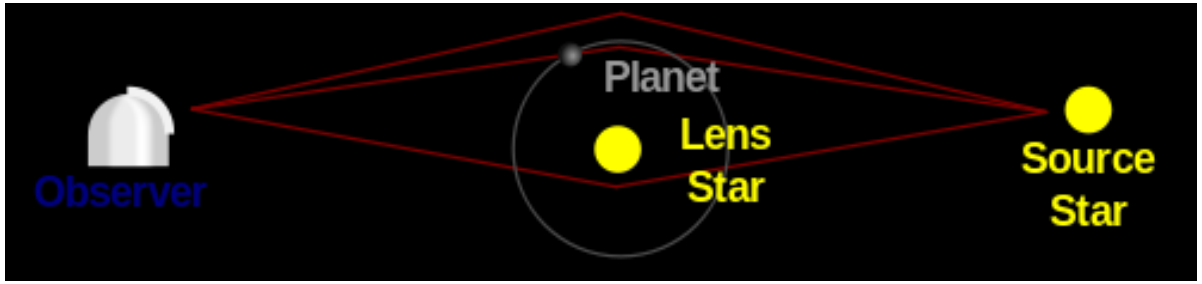
Radius

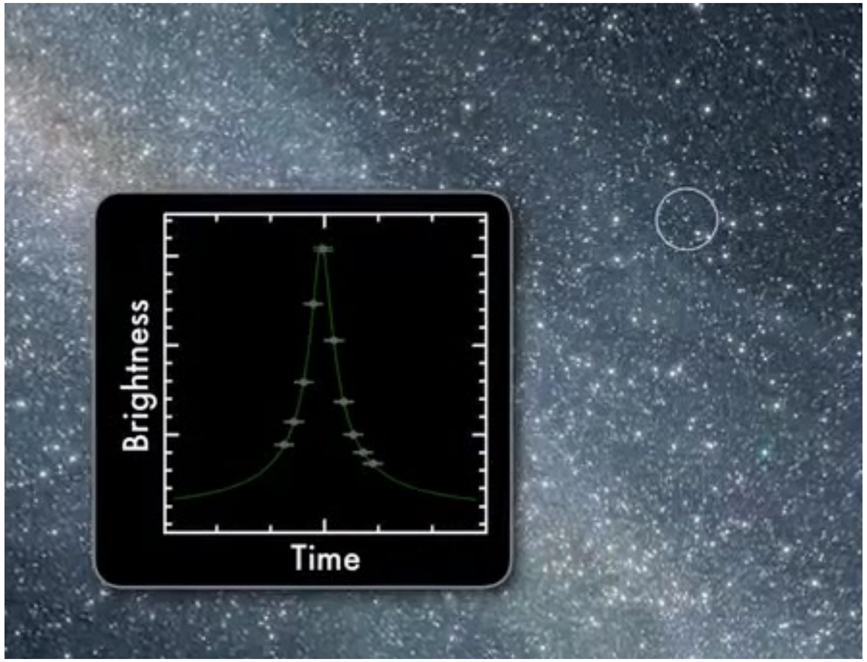


Mass



Microlensing



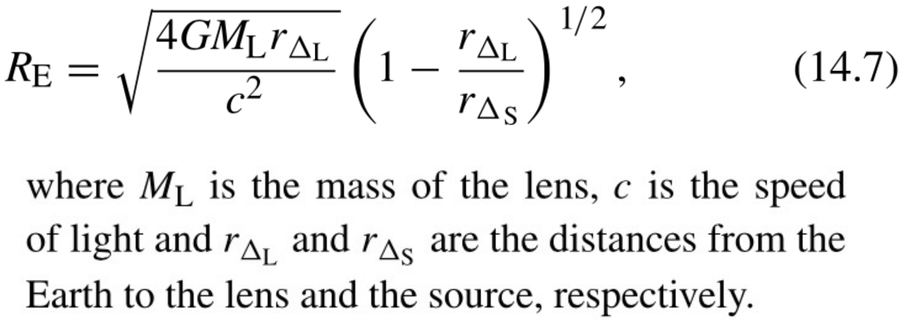


https://www.youtube.com/watch?v=FHh0Qx7LPJY

Gravitational microlensing occurs when the gravitational field of a lens star acts like a lens, magnifying the light of a distant background star. This effect occurs only when the two stars are almost exactly aligned.

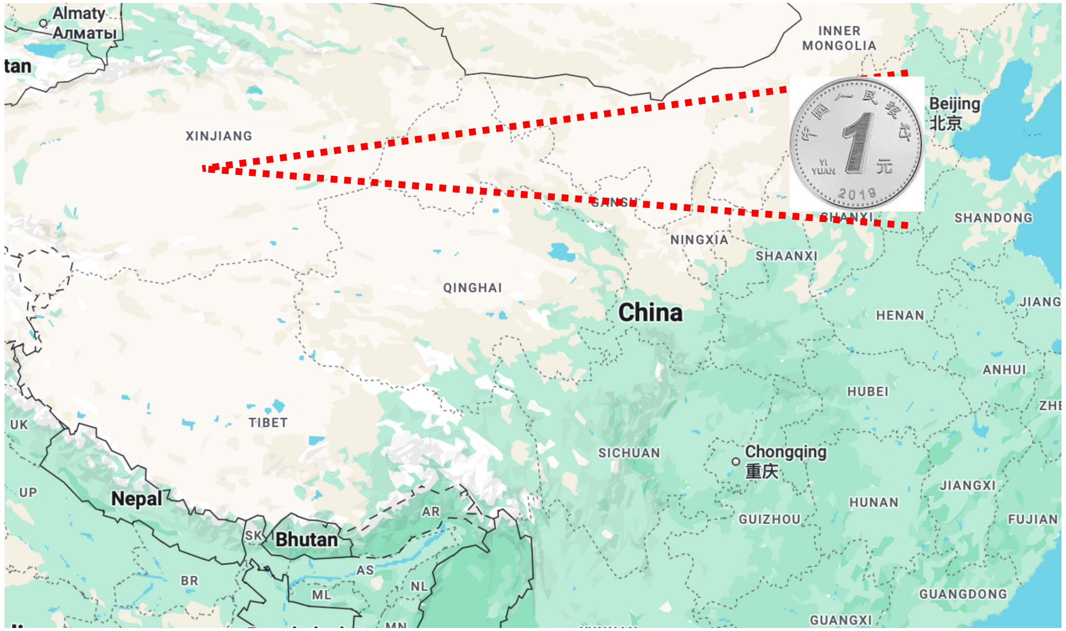
Microlensing

- Only observable when the source and the lens are very well aligned (separation on the order of the Eisenstein radius)
- Typical Eisenstein radius: 1 mas, or 5e-9 radian
- Need to monitor millions of stars to identify microlensing events



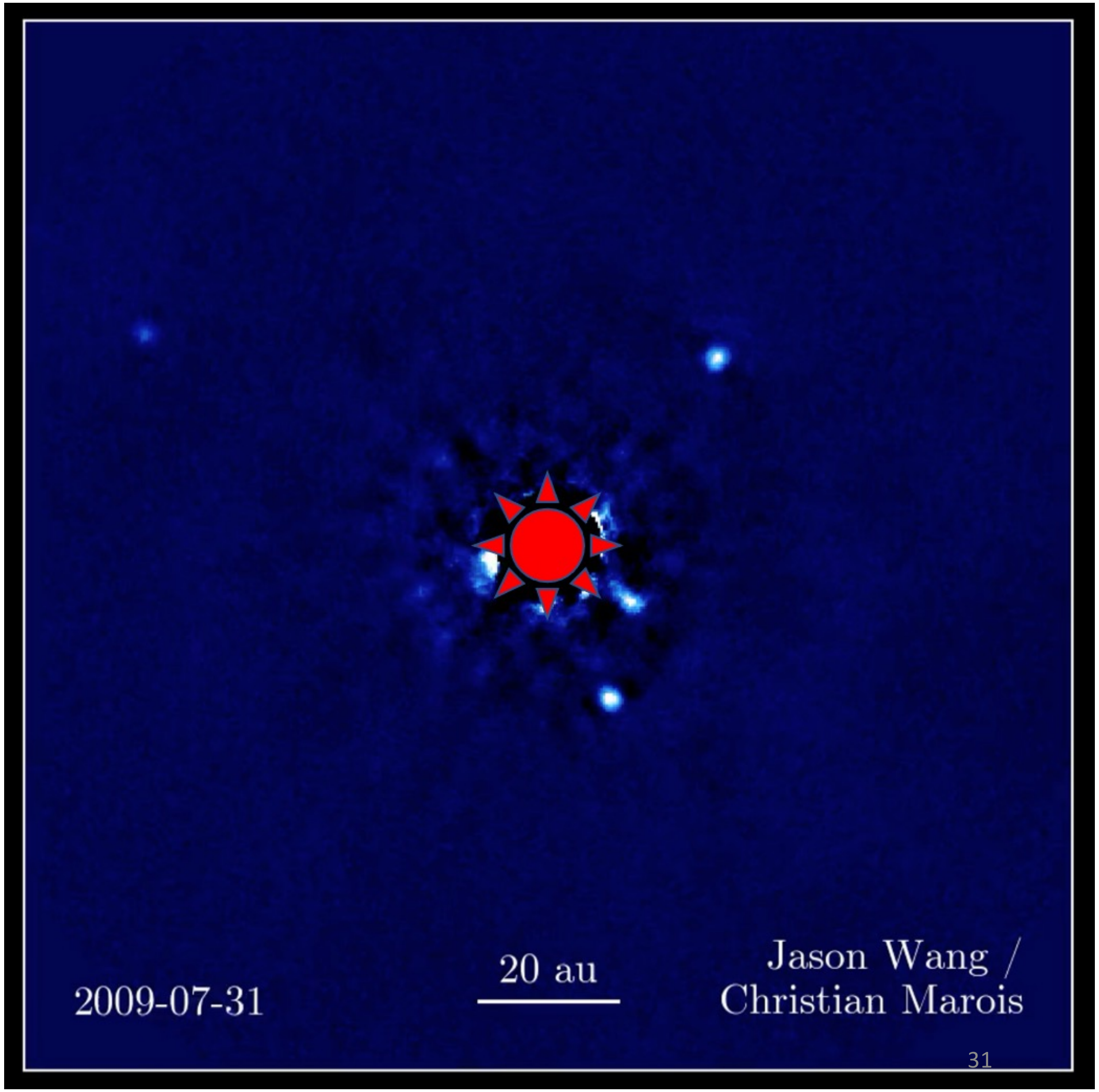
$$R_{\mathrm{E}} = \sqrt{\frac{4GM_{\mathrm{L}}r_{\Delta_{\mathrm{L}}}}{c^2}} \left(1 - \frac{r_{\Delta_{\mathrm{L}}}}{r_{\Delta_{\mathrm{S}}}}\right)^{1/2}, \qquad (14.7)$$

where M_{L} is the mass of the lens, c is the speed of light and $r_{\Delta_{\mathrm{L}}}$ and $r_{\Delta_{\mathrm{S}}}$ are the distances from the Earth to the lens and the source, respectively.



Direct Imaging

Contrast between the planet and the host star?



Direct Imaging

Planck's law

https://en.wikipedia.org/wiki/Planck%27s_law

Stefan–Boltzmann law

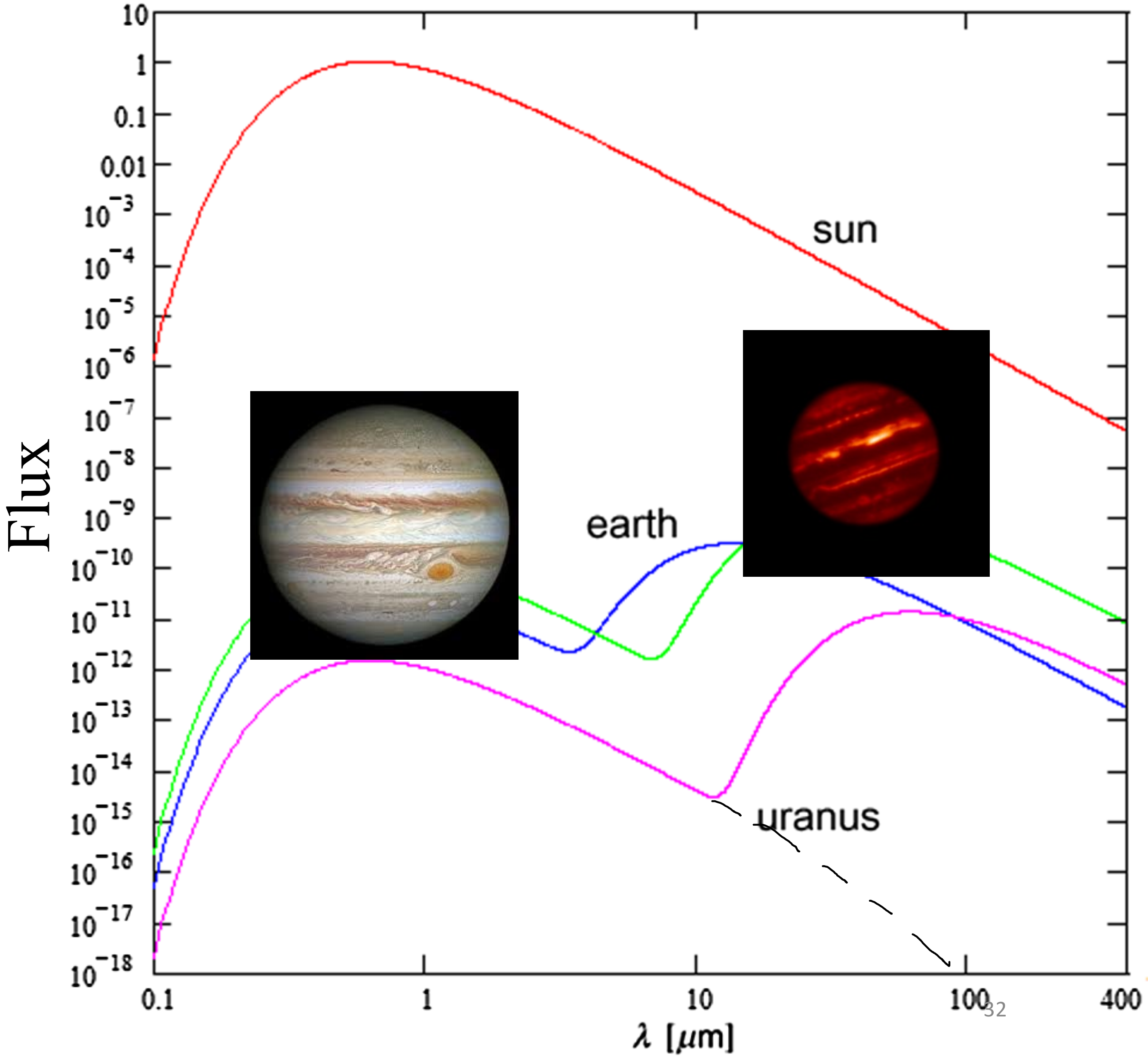
https://en.wikipedia.org/wiki/Stefan–Boltzmann_law

Rayleigh-Jeans law

https://en.wikipedia.org/wiki/Rayleigh–Jeans_law

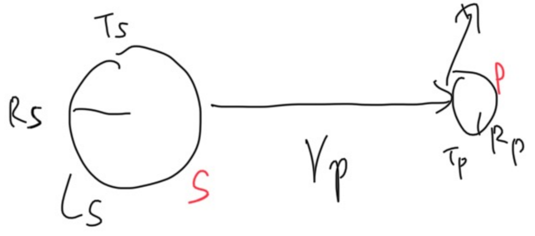
Wien's displacement law

https://en.wikipedia.org/wiki/Wien's_displacement_law



Credit: NASA

Brightness ratio between a star and the earth



reflected light

$$L_p = \underbrace{\frac{L_s}{4\pi r_p^2}}_{\text{flux at plaket location}} \times \overset{\text{cross section}}{\underline{\pi R_p^2}} \times \underline{A_p} \ \text{Albedo (angle dependent)}$$

Brightness ratio at a given wave length. λ $\left.\frac{L_s}{L_p}\right|_\lambda$

$$\frac{L_p}{L_s} = \frac{\pi R_p^2}{4\pi r_p^2} A_p$$

$R_p = 5\times10^{-5}$ AU

$r_p = 1$ AU

$$= \frac{(5\times10^{-5})^2}{4\times1^2} A_p = 6\times10^{-10} A_p \sim 10^{-10}$$

thermal emission (blackbody)

$$L_{s,\lambda} = \underbrace{4\pi R_s^2}_{\text{total surface area}} \times \underbrace{B_\lambda(T_s)}_{\text{planck F.}} \times \pi$$

$$L_{p,\lambda} = 4\pi R_p^2 \times B_\lambda(T_p) \times \pi$$

Blackbody radiation peaks at λ_{peak} (Wien's displacement law)

$$\lambda_{peak} = \frac{b}{T} \quad (\lambda_{peak,s} < \lambda_{peak,p})$$

If we work at $\lambda \gg \lambda_{peak,p}$, $B_\lambda(T) \sim \frac{2ck_BT}{\lambda^4}$, Rayleigh Jeans

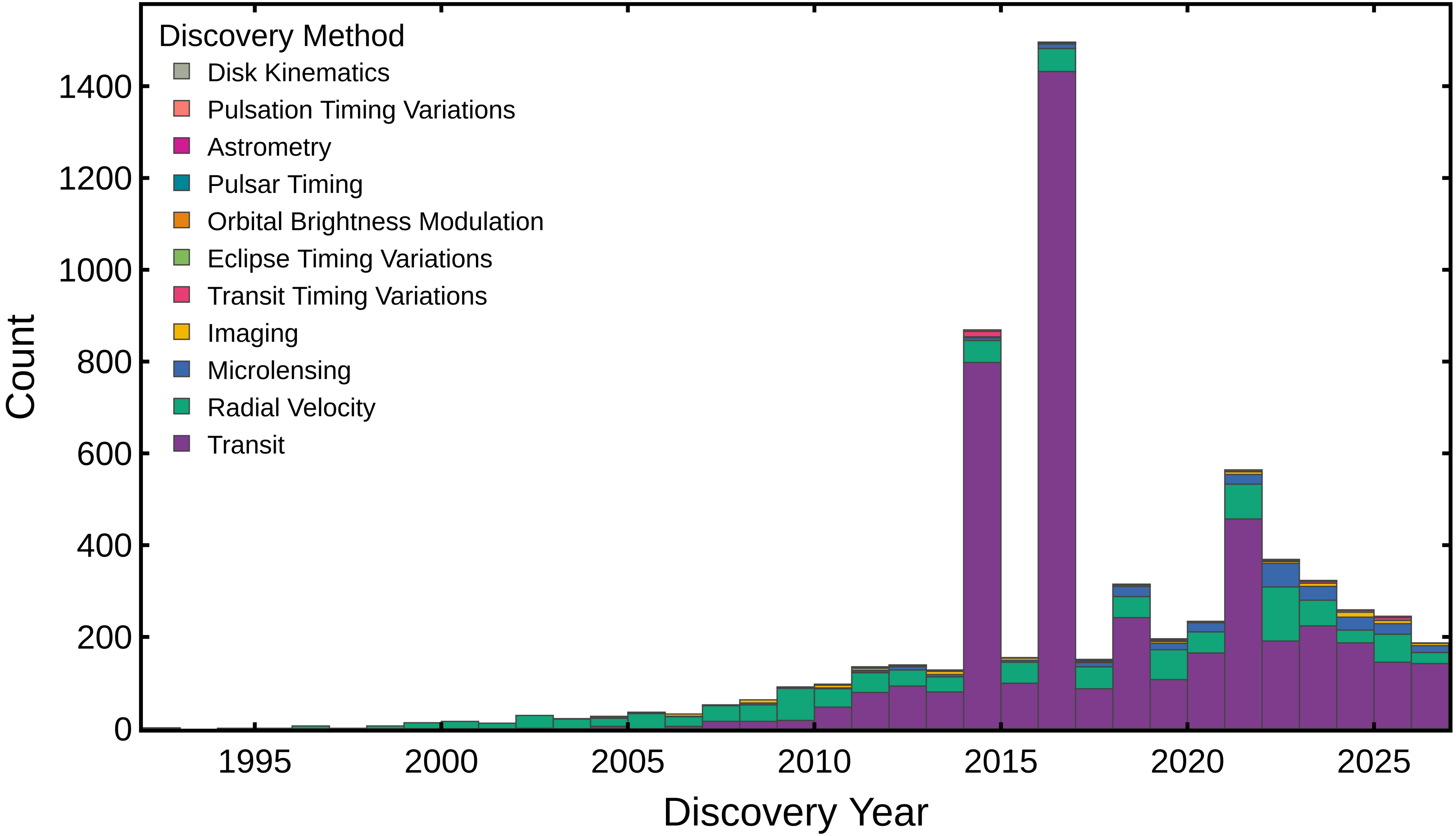
$$\frac{L_{p,\lambda}}{L_{s,\lambda}} = \frac{4\pi R_p^2 B_\lambda(T_p)}{4\pi R_s^2 B_\lambda(T_s)} = \left(\frac{R_p}{R_s}\right)^2 \frac{T_p}{T_s} = \left(\frac{1}{100}\right)^2 \frac{300K}{6000K} = 5\times10^{-6}$$

Observations of Exoplanets

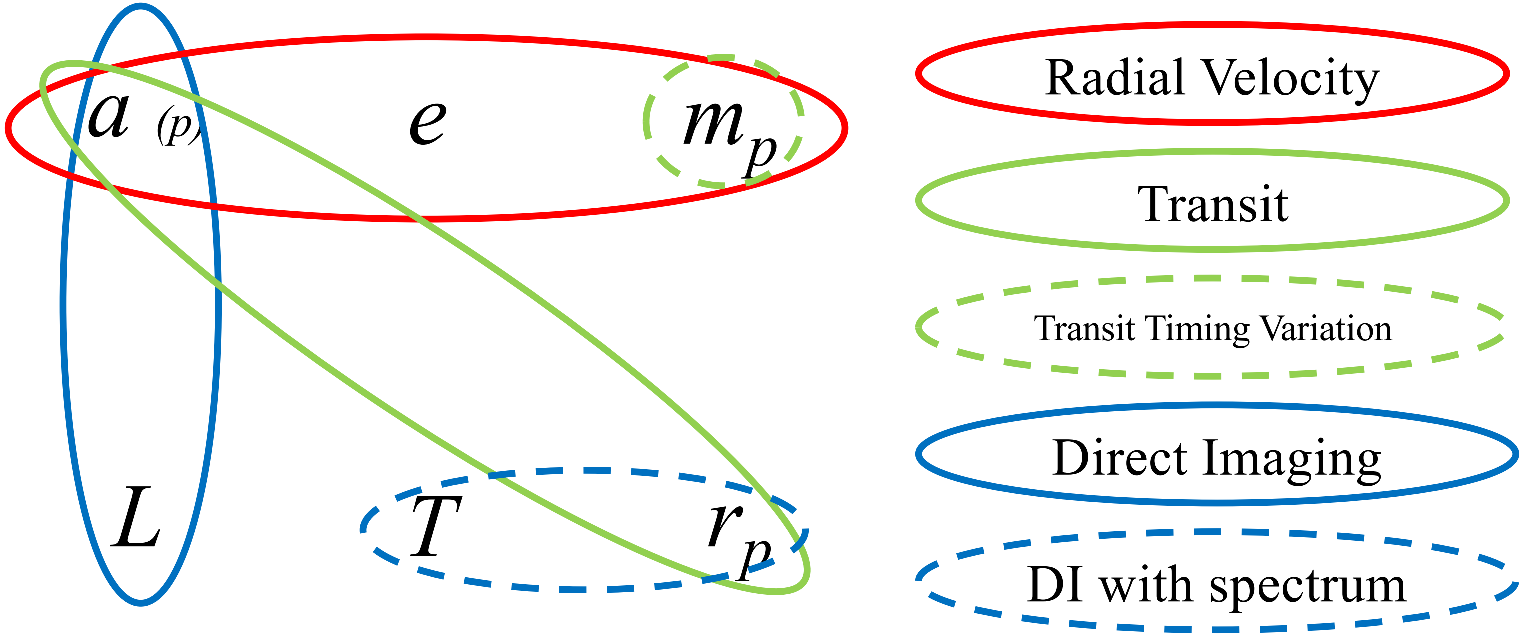
14.3

Counts vs Discovery Year

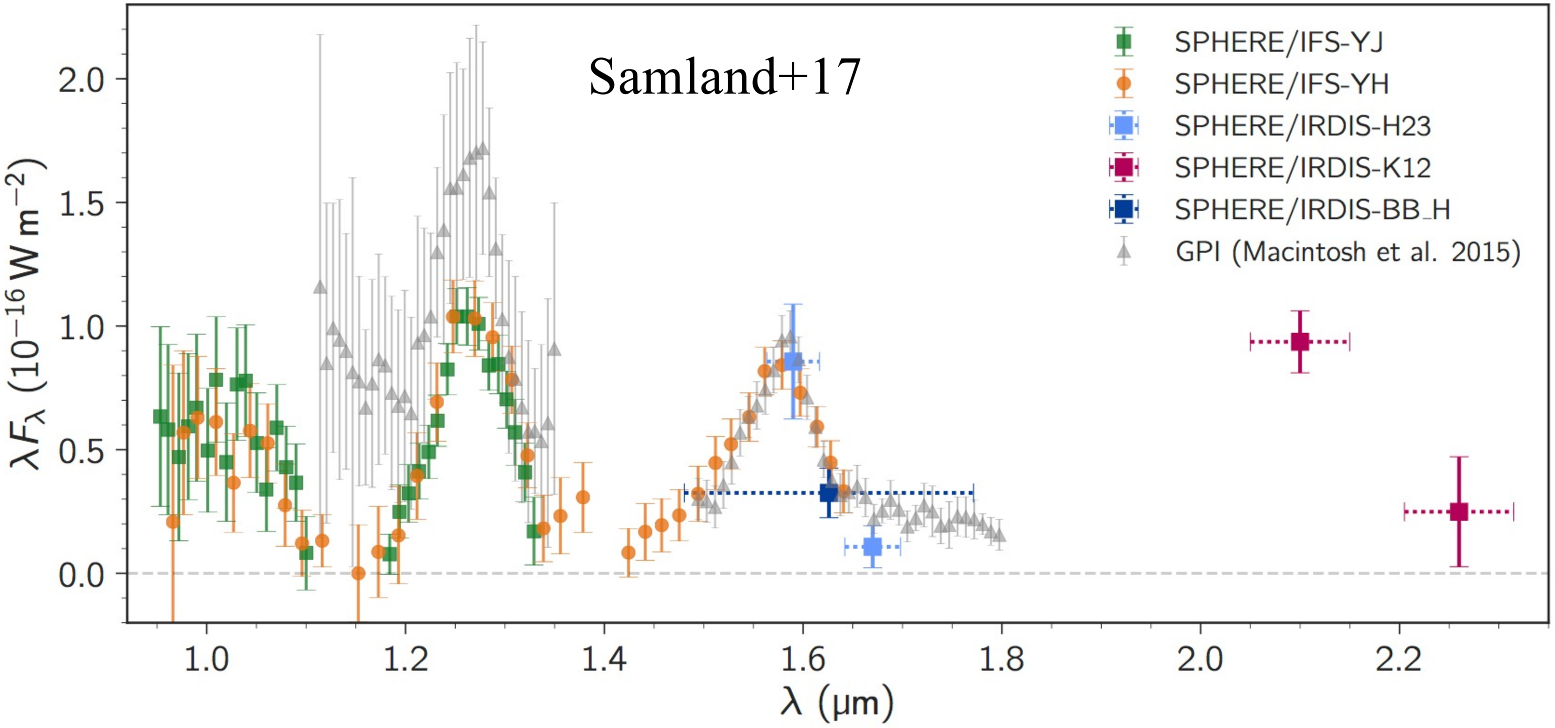
exoplanetarchive.ipac.caltech.edu, 2026-04-23



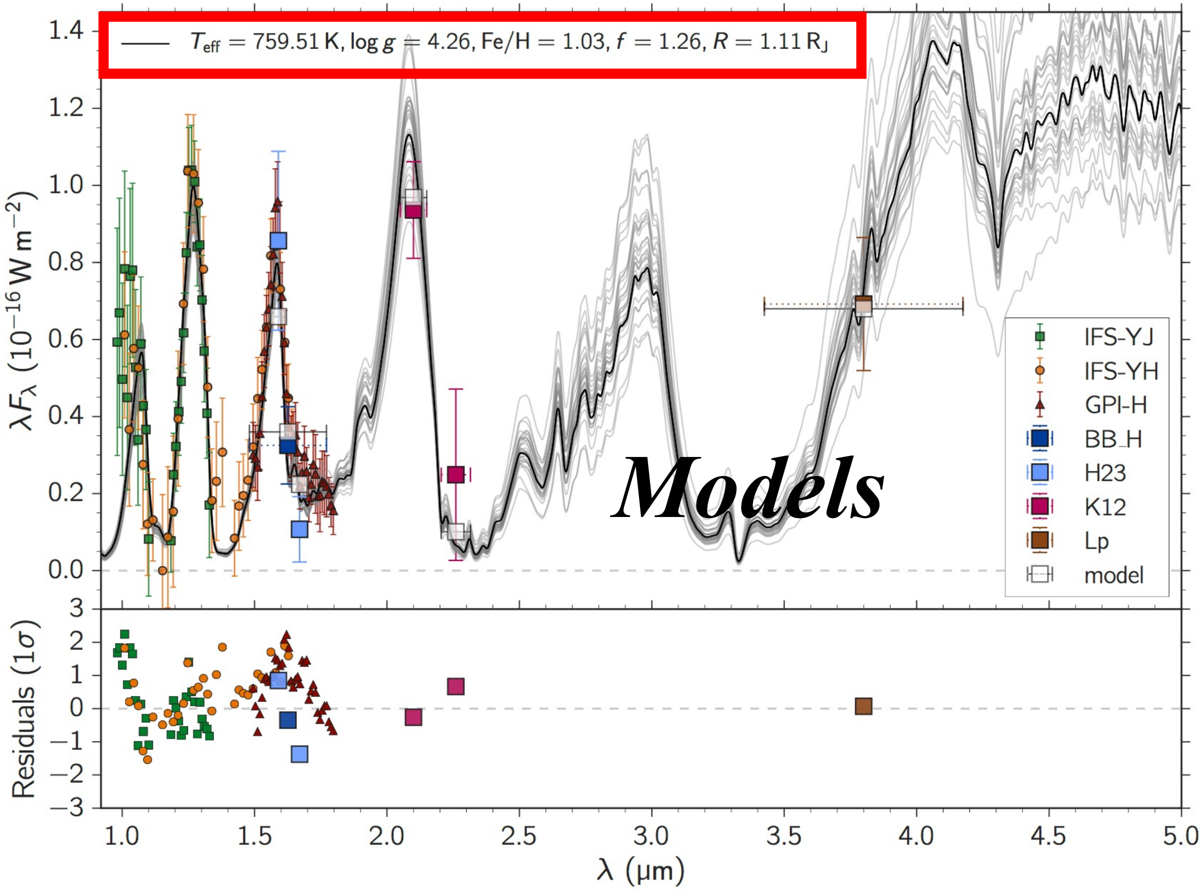
Basic Properties



Spectrum of an exoplanet: from direct imaging

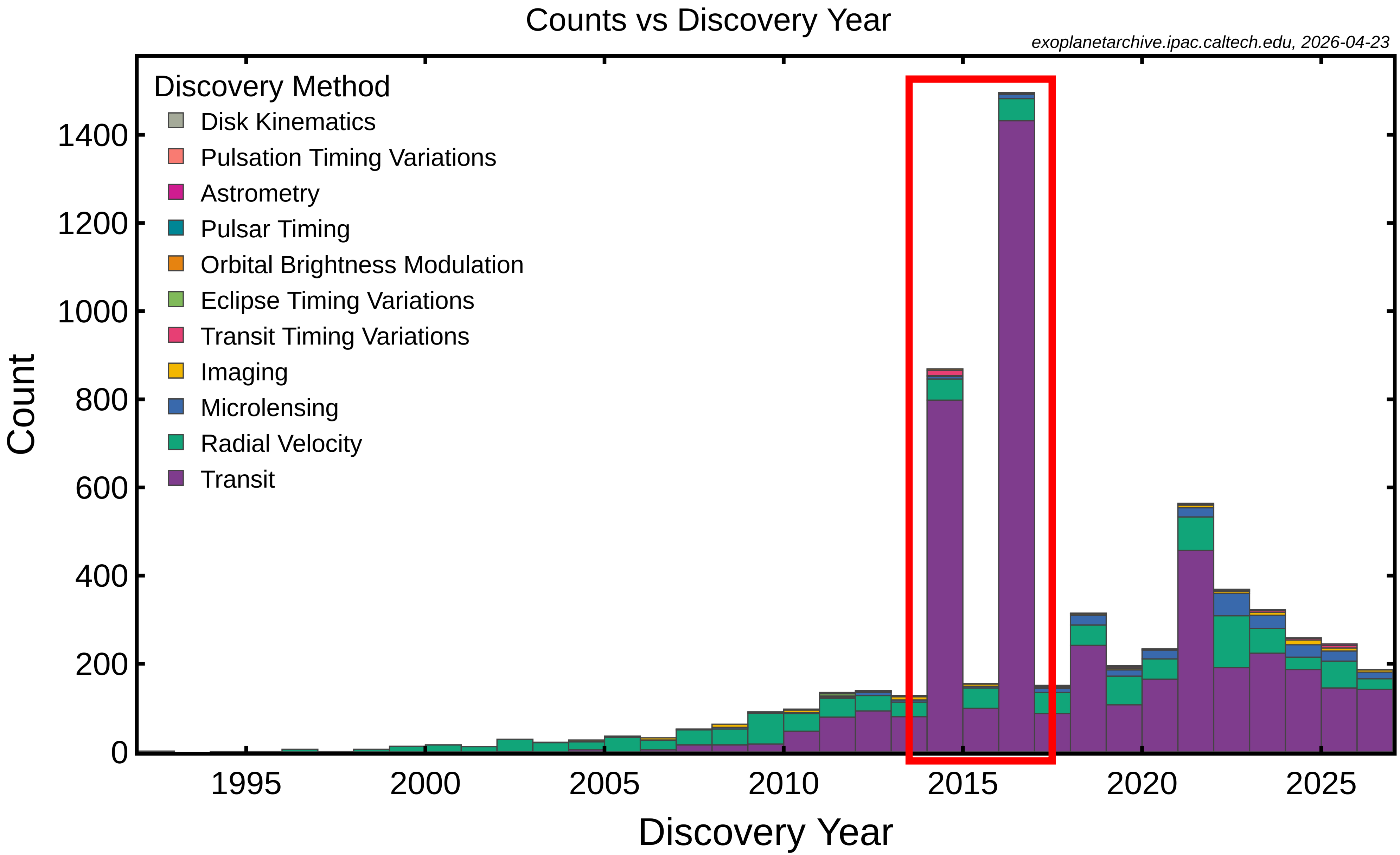


Spectrum of an exoplanet → Atmosphere properties



Models

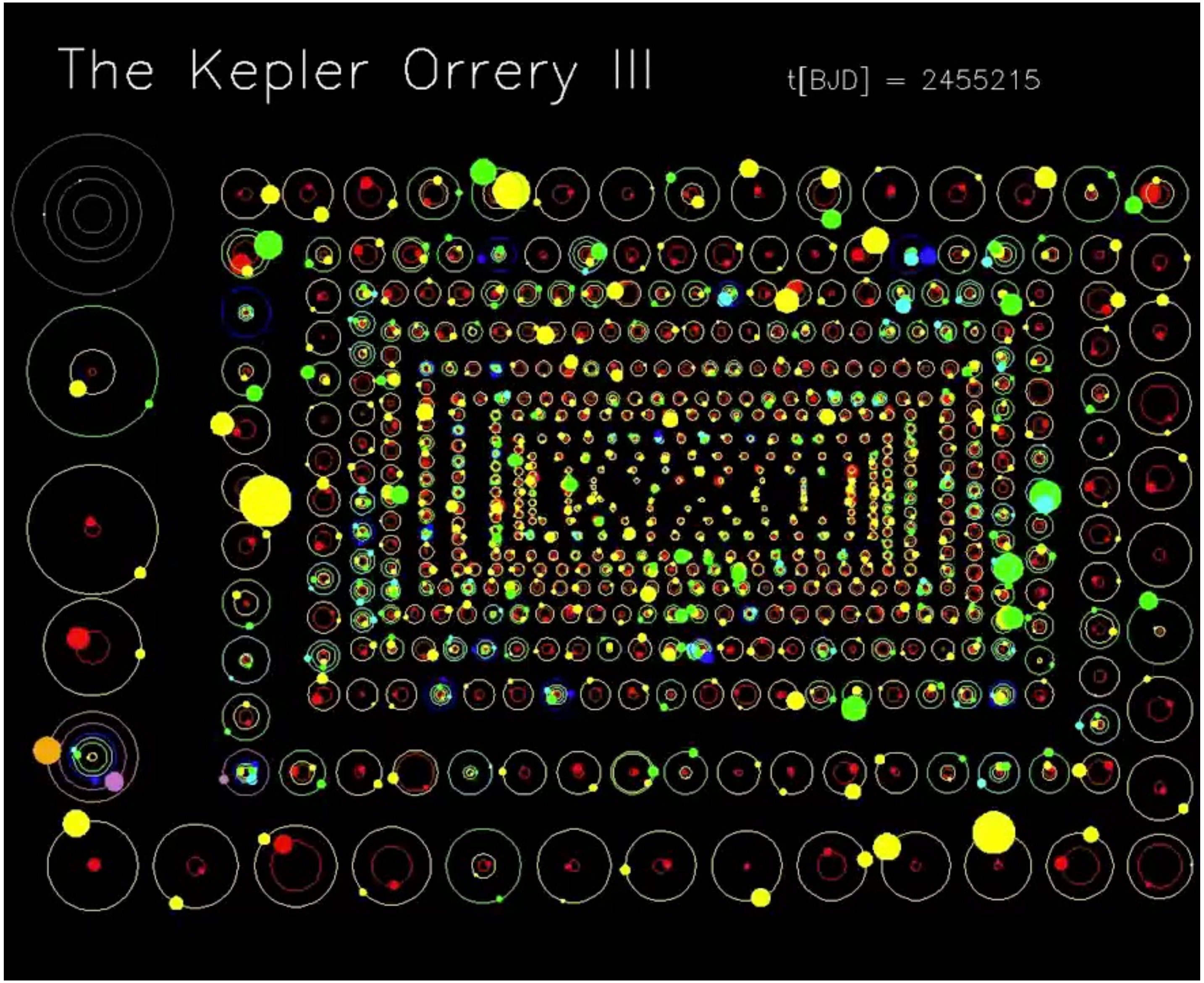
Samland+17


Counts vs Discovery Year
exoplanetarchive.ipac.caltech.edu, 2026-04-23
Discovery Method
Disk Kinematics
Pulsation Timing Variations
Astrometry
Pulsar Timing
Orbital Brightness Modulation
Eclipse Timing Variations
Transit Timing Variations
Imaging
Microlensing
Radial Velocity
Transit
Count
0
200
400
600
800
1000
1200
1400
1995
2000
2005
2010
2015
2020
2025
Discovery Year

Kepler Mission

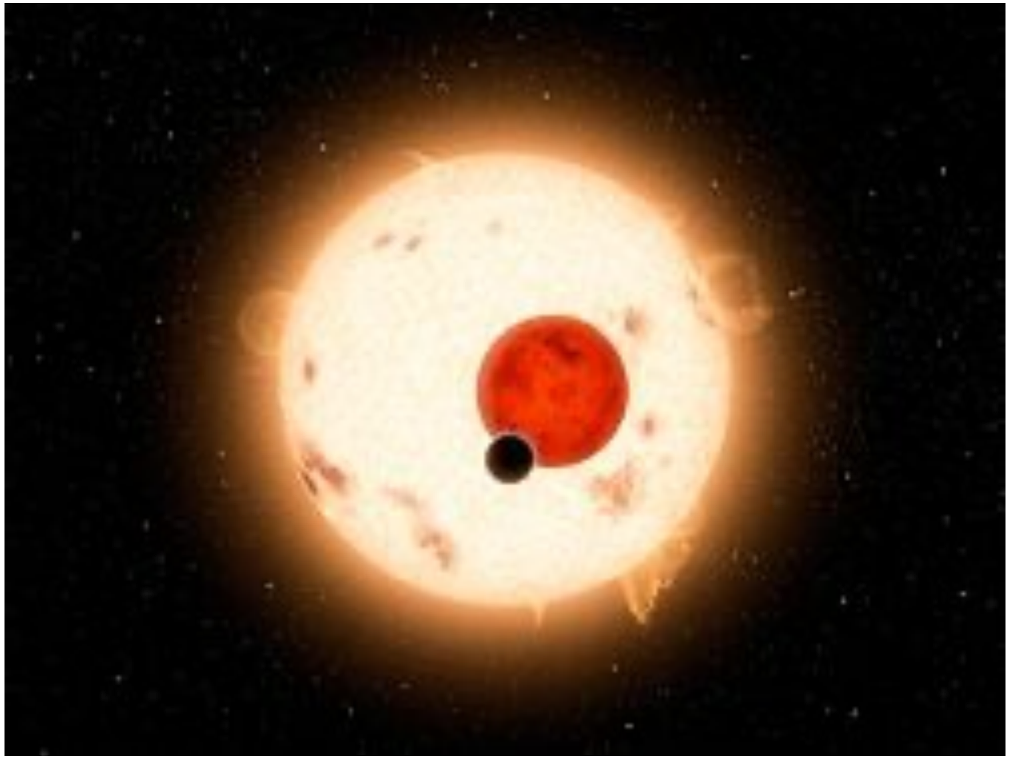
https://www.youtube.com/watch?v=3Wxd3fDFmO4

Some Kepler Discoveries



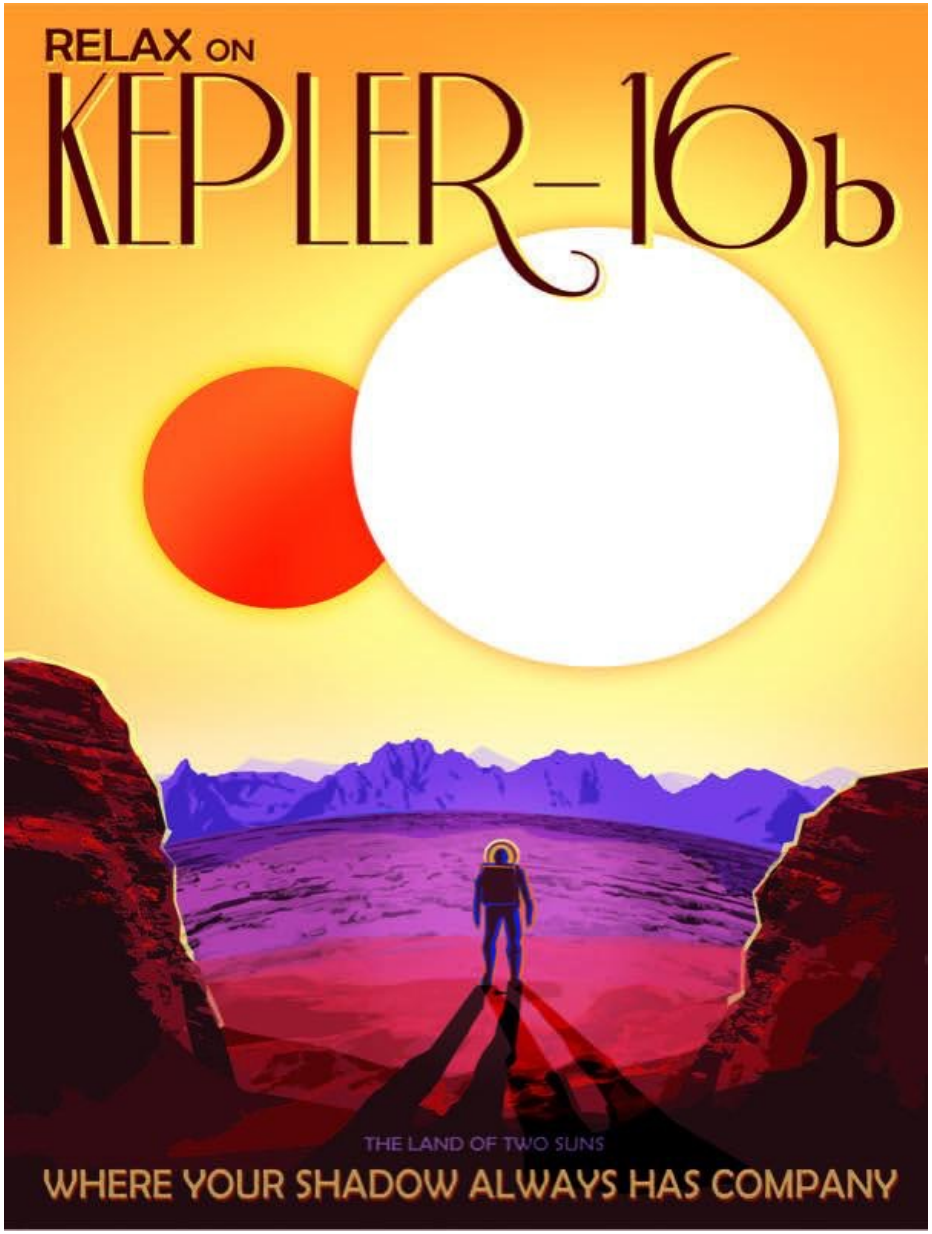
D. Fabrycky

Example of Kepler's discoveries: Circumbinary Planets



https://en.wikipedia.org/wiki/Kepler-16

Doyle+11

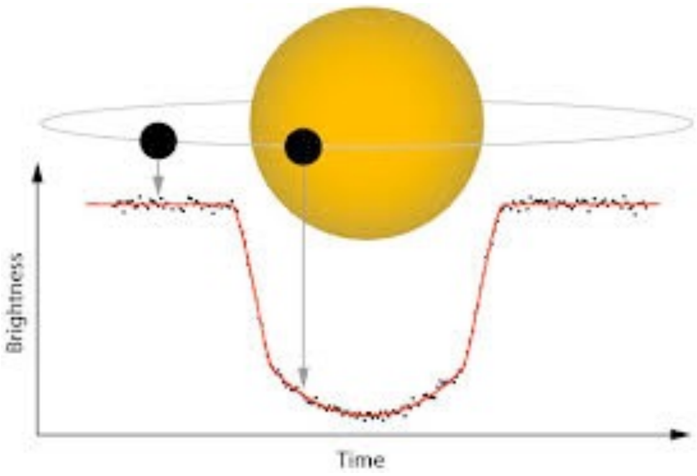


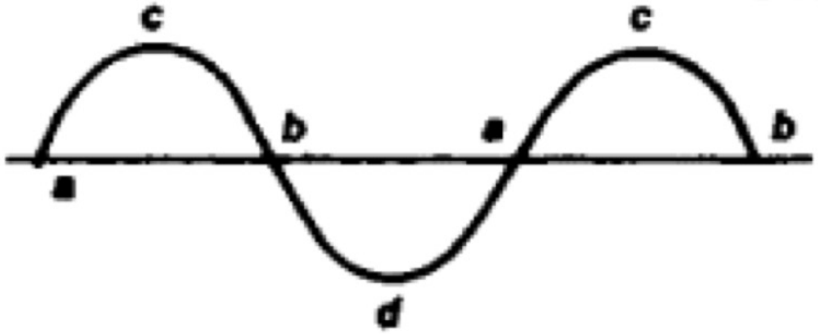
Exoplanet Properties and Demographics

14.4 & 14.5

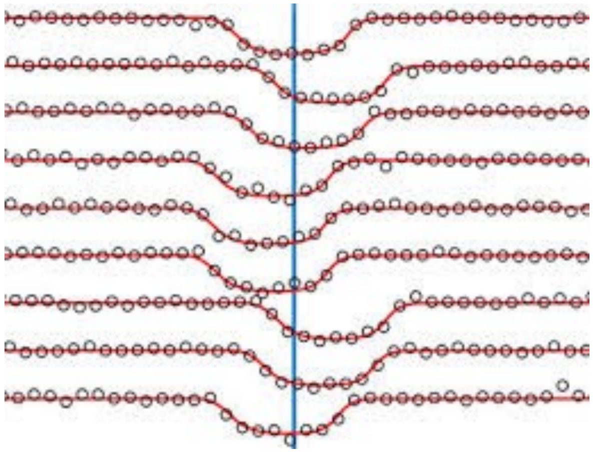
Density of exoplanets: Need both radius and mass

Example: ***Radius*** ***Mass***





RV

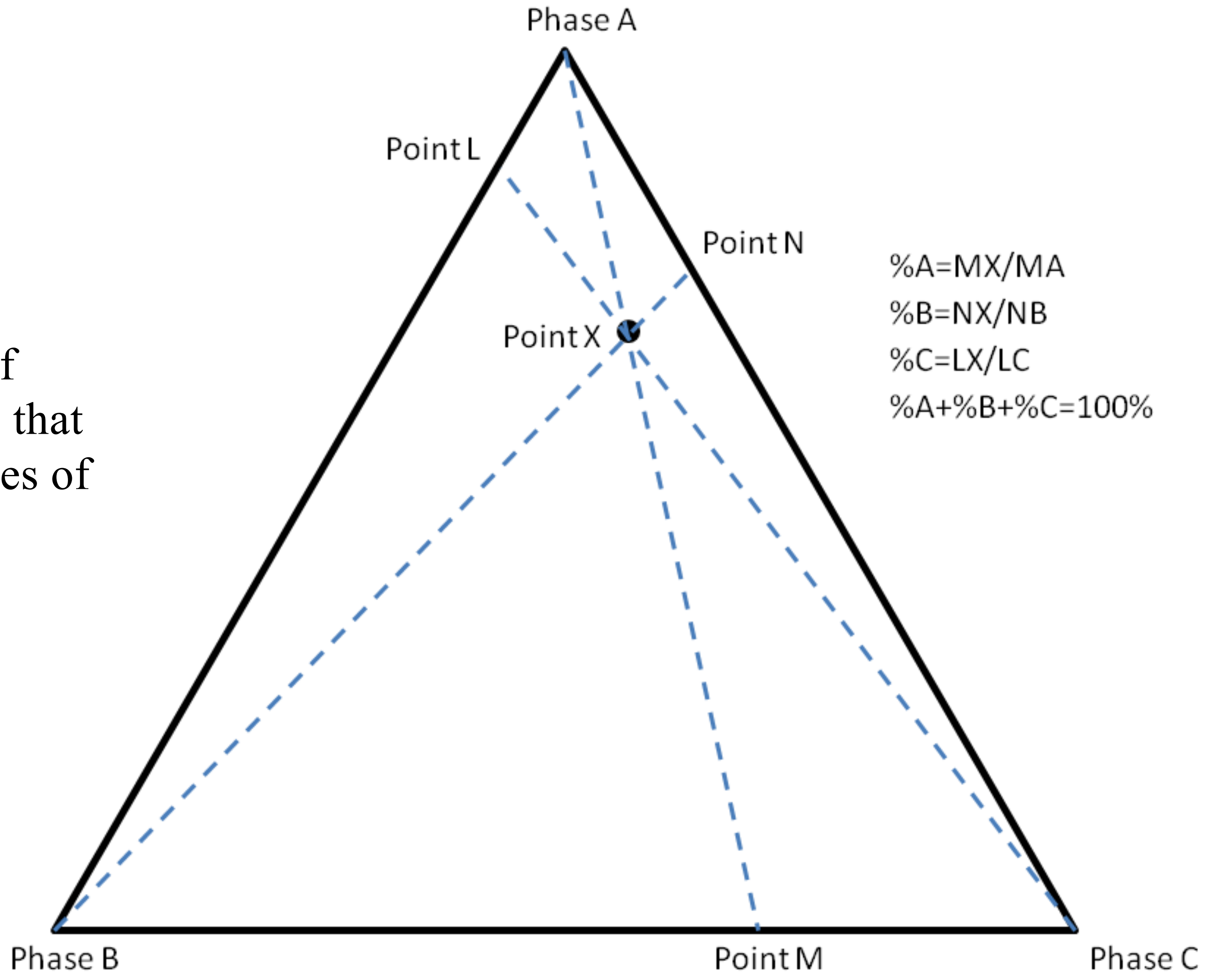


TTV

Mass-Radius Relationship

triangular representations of the compositions of objects that can be expressed by mixtures of three components.

https://en.wikipedia.org/wiki/Ternary_plot



%A=MX/MA

%B=NX/NB

%C=LX/LC

%A+%B+%C=100%

Mass-Radius Relationship

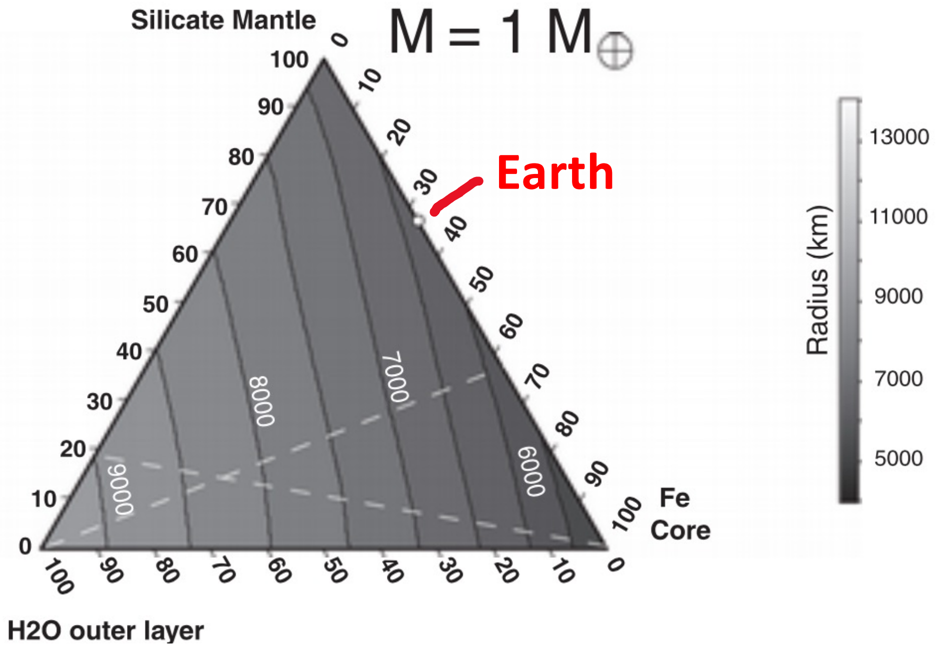
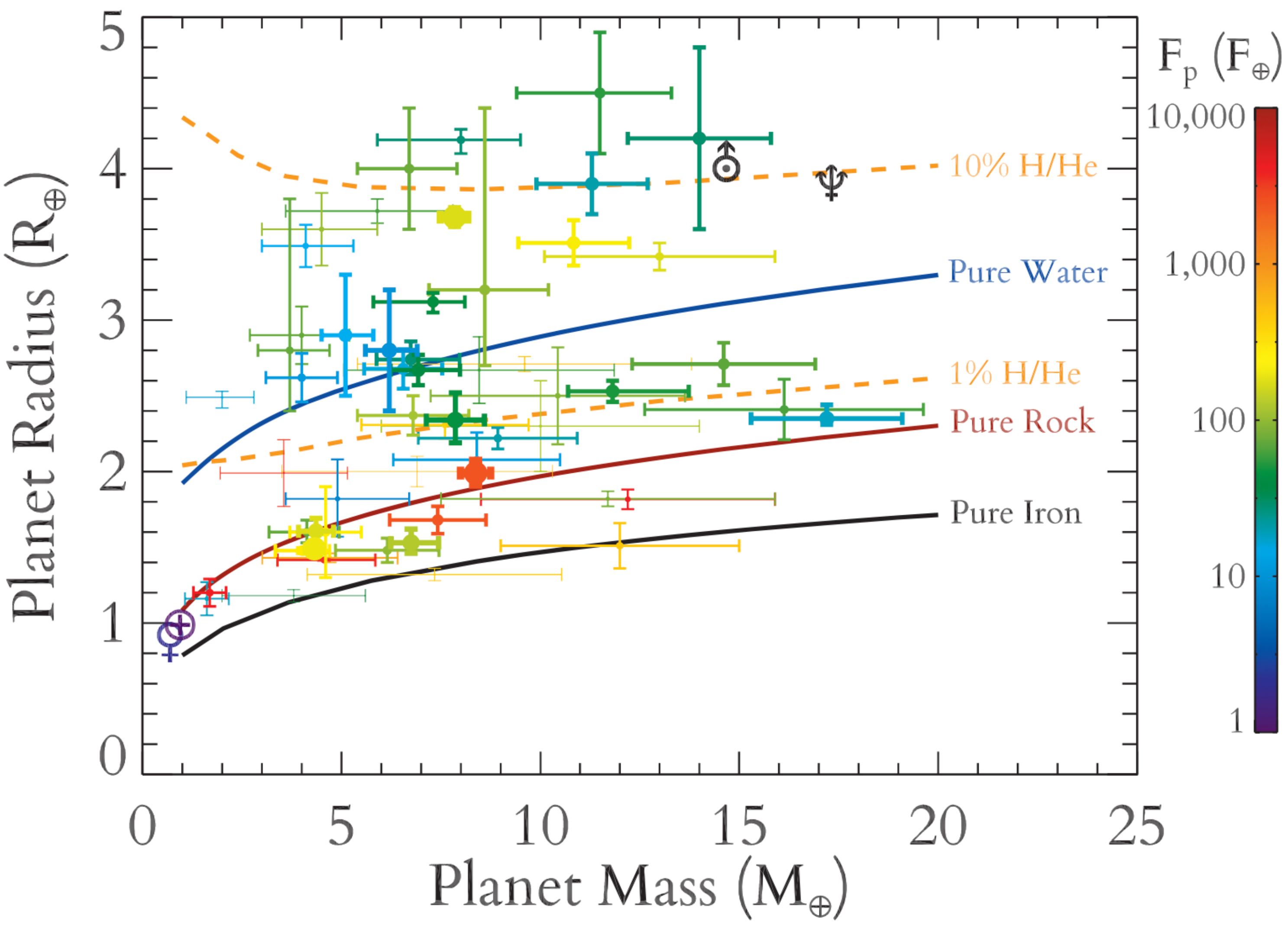


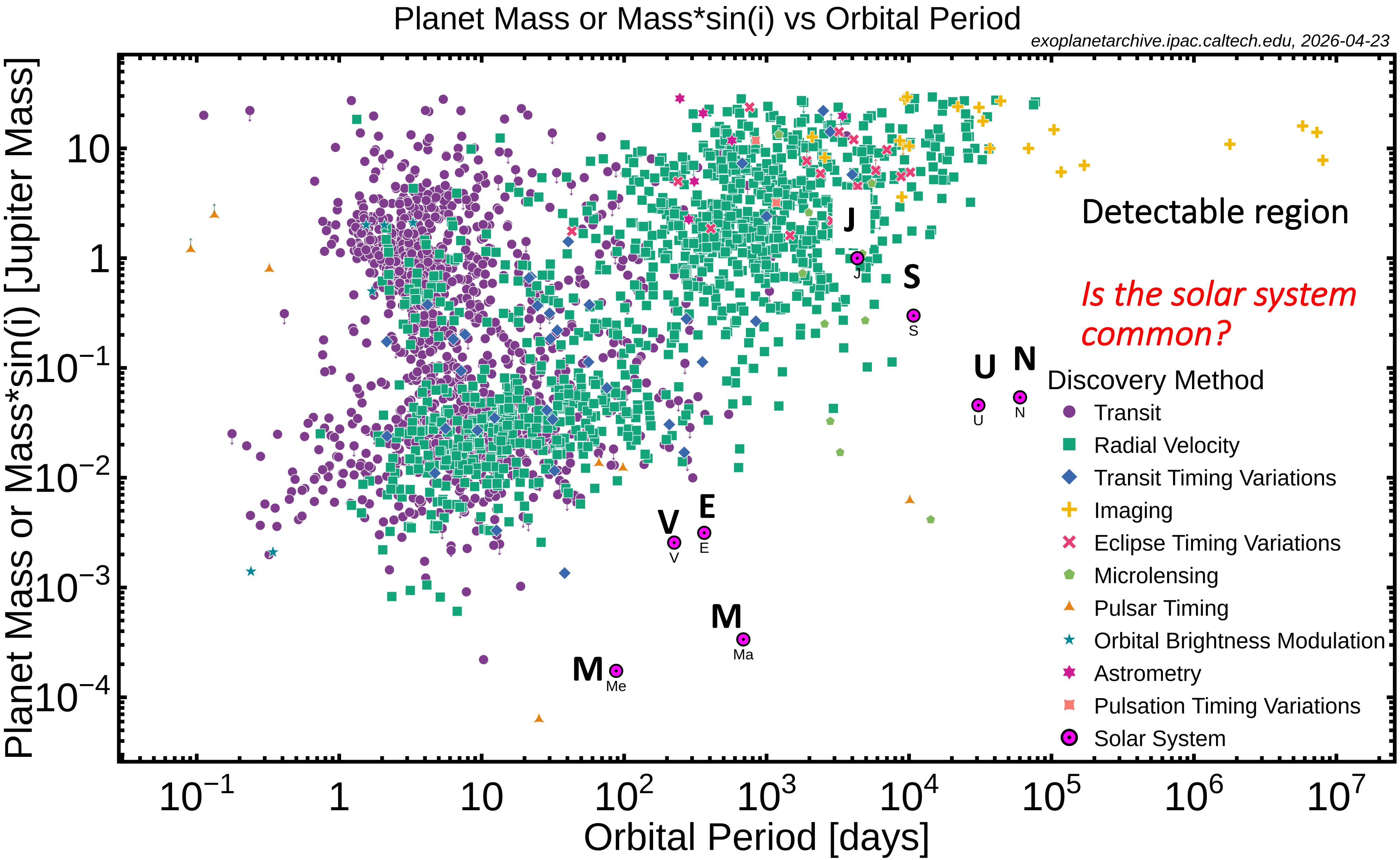
Figure 14.22 Ternary diagrams showing the relationship between composition and radius for 1 $M_\oplus$ and 5 $M_\oplus$ planets. *Solid curves* representing constant radius are shown in *black* at increments of 500 km. Different mixtures of the three most likely end-member components (iron cores, silicate mantles and H_2O outer regions) yield planets of different sizes. Each point in the ternary diagrams depicts a unique composition with a corresponding radius shown by the *shade of gray color*. The three vertices correspond to pure compositions of H_2O, iron or silicates, and the opposite sides of the triangle correspond to 0% of that end member. Thus, the side that connects Fe core and silicate mantle represents waterless planets. Earth's composition is shown by a *circle* essentially on this line in the 1 $M_\oplus$ diagram. Planets that formed in disks of solar nebula composition that are composed of all substances that condense above any specified temperature (Fig. 15.5), or mixtures of materials that condense at a range of temperatures, lie above both of the *dashed lines*. (Courtesy Diana Valencia)

Mass-Radius Relationship



Stellar irradiation

Plate 14.24



Detectable region

Is the solar system common?